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Past Performance Evaluation in Repeated Procurement: A Simple Model of Handicapping



Ministero  
dell'Economia  
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## Abstract

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When procurement contracts are awarded through competitive tendering, participating firms commit ex ante to fulfill a set of contractual duties. However, selected contractors may find it profitable to renege ex post on their promises by opportunistically delivering lower quality standards. In order to deter ex post moral hazard, buyers may use different strategies depending on the extent to which quality dimensions are contractible, that is, verifiable by contracting parties and by courts. We consider a stylized repeated procurement framework in which a buyer awards a contract over time to two firms with different efficiency levels. If the contractor does not deliver the agreed level of performance the buyer may handicap the same firm in the next competitive tendering. We prove that, under complete information, extremely severe handicapping never induces the contractor to fulfill the quality requirement, rather the buyer finds it optimal to punish the opportunistic firm so as to make the pool of competitors more alike. In other words, when opportunistic behavior arises, the buyer should use handicapping to "level the playing field".

Repeated Procurement; Handicapping; Relational Contracts; Stick and Carrot Strategy

*JEL Classification Numbers: C 73, D82, D44, H57, K12, L14*

When procurement contracts are awarded through competitive tendering participating firms commit ex ante to fulfill a set of contractual duties. However, selected contractors may find it profitable to renege ex post on their promises by opportunistically delivering lower quality standards. In order to deter ex post moral hazard, when delivered quality is verifiable by a third part then a standard principal-agent model applies and an explicit contract can be specified ex ante. However, there exist some goods or services whose quality is hard to verify, for example the services essentially based on a high human capital component like IT and consulting. When the contractor's performance consists essentially in the provision of human capital the buyer may find it hard, if not impossible, to prove objectively whether the contractor has exactly complied with the contractual duties. When quality is not verifiable a formal contract cannot be enforced by a third party, therefore it needs to be self-enforcing in order to be effective.

Since procurement contracts are repeatedly awarded over time, reputation mechanisms may play a crucial role in providing dynamic incentives for contractors to fulfill contractual clauses. A special form of reputation mechanism is to award a certain score to a participating firm based on its past performance. For instance, the U.S. Federal Acquisition Regulation prescribes that "past performance should be an important element of every evaluation and contract award for commercial items. Contracting officers should consider past performance data from a wide variety of sources both inside and outside the Federal Government[...]. (FAR, 12.206)".

# 1. Introduction

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In a context of complete information and observable but non verifiable quality we allow the buyer to handicap a firm that behaved opportunistically in the past. We consider a stylized repeated procurement framework in which a buyer awards a contract over time to two suppliers with different efficiency levels. If the firm did not provide satisfactory quality levels in a previously awarded contract, the buyer may reduce at her discretion the score assigned to the tender submitted by that firm in the future competitive tendering. We prove that extremely severe handicapping never induces the contractor to fulfill the quality requirement, rather the buyer finds it optimal to punish the opportunistic firm so as to make the pool of competitors more alike. In other words, when opportunistic behavior arises, the buyer should use handicapping to "level the playing field" .

In particular, we set up an infinitely repeated game whose constituent (static) game is composed of three stages. At the first stage a simplified version of the sealed-bid competitive tendering takes place: the buyer requires fulfillment of a minimal quality standard and two fully informed heterogeneous firms bid over price only. At the second stage, once the contract is awarded, the contractor chooses the quality. At the last stage the buyer observes the effective quality and decides whether to handicap.

We allow the buyer and the contractor to use a single period punishment. When no cheating on quality is observed no handicap is applied; otherwise the buyer handicaps the opportunistic contractor only in the next competitive tendering. On the other hand, the firm does not cheat if no cheating and handicap has occurred until that moment, otherwise it delivers zero quality only for one period.

This paper shows that the optimal strategy for the buyer is to impose in the next competitive tendering a handicap equal to the efficient firm's cost advantage. In this scenario the bidders are symmetric and get the same score in the competitive tendering. Given a tie breaking rule awarding the contract to the efficient firm, a sufficiently patient efficient contractor prefers not to shrink rather than win the next competitive tendering at a lower price. An extremely harsh handicap (that is equivalent to exclusion from the next competitive tendering) is not an optimal strategy for the buyer for two reasons. First, it implicitly awards the contract to the less efficient firm that will bid less aggressively and always deliver zero quality. Second, given this reaction by the less efficient firm, the efficient contractor prefers to provide zero quality, lose the tendering for one period and then be reawarded the contract when the less efficient firm is handicapped. On the other hand, a handicap lower than the firm's cost advantage is not an optimal strategy for the buyer as well.

In this scenario the efficient firm still wins the competitive tendering by gaining a positive profit, therefore it has a lower incentive to deliver the required quality. In particular, the lower the handicap the less aggressive is the equilibrium bid and then lower is the procurer's utility.

Our paper shows that repeatedly awarded procurement contracts in which unverifiable quality dimensions are relevant can be reinterpreted as relational contracts between a buyer and a contractor that is threatened by a potentially less efficient competitor. Relational contracts pioneered by MacLeod and Malcomson (1989) and refined by MacLeod (2003) and Levin (2003) consider non-verifiable performance dimensions.

Since such contracts are not legally enforceable, they need to be self-enforcing in order to be effective. These papers set up a infinitely repeated interaction between a principal and an agent by assuming that the performance of the latter is non-verifiable. The main message is that a wage scheme composed of a fix and a discretionary payment depending on the performance usually characterizes an optimal self-enforcing contract. All these papers employ a trigger strategy as in Abreu (1988) in which the discretionary payment is used by the principal to punish the cheating agent with the worst equilibrium outcome. We do not introduce a direct punishment strategy as in MacLeod and Malcomson (1989), MacLeod (2003) and Levin (2003).

Our punishment is indirect in the sense that it does not consist in a direct cost in the contractor's utility, rather we allow the buyer to alter the subsequent competitive tendering by reducing the score of the opportunistic contractor. A further contribution of this paper is that our punishment lasts only one period ("stick and carrot"). Such a strategy sounds more realistic in procurement markets where, unless a serious wrongdoing like corruption or bribery is committed, a buyer cannot resort to trigger strategies thus keeping any form of punishment alive from one specific moment onwards<sup>1</sup>.

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<sup>1</sup> Permanent punishments for serious wrongdoing are introduced in the *Bidding Documents for Procurement of Goods* prepared by the World Bank to be used for the procurement of goods through International Competitive Bidding (ICB). These guidelines are applied in projects that are financed in whole or in part by the World Bank. They incorporate the *Guidelines for Procurement under IBRD Loans and IDA Credits*. These documents prescribe that: "...The Bank: ... (c) will cancel the portion of the loan allocated to a contract if it determines at any time that...the beneficiary of the loan engaged in corrupt, fraudulent, collusive, or coercive practices during the procurement or the execution of that contract... (d) will sanction a firm or individual, including declaring ineligible, either indefinitely or for a stated period of time, to be awarded a Bank-financed contract if it at any time determines that the firm has...engaged in corrupt, fraudulent, collusive, coercive or obstructive practices...in executing, a Bank-financed contract..."

Our paper bears some ingredients from MacLeod (2003) that sets up a repeated framework in which the performance evaluation depends on the correlation between the principal's and the agent's beliefs. MacLeod, in fact, assumes that the agent's beliefs about his performance are correlated with the principal's ones. Our paper captures the case of perfect correlation.

To the best of our knowledge papers strictly related to past performance evaluation in repeated procurement are Kim (1998), Doni (2006) and Spagnolo and Calzolari (2006). Kim (1998) and Spagnolo and Calzolari (2006) assume an extreme handicap since the buyer is allowed to debar the opportunistic contractor from the subsequent competitive tenderings. All these papers model a repeated game in which the level of handicapping is exogenous, whereas we find the credible level of handicapping characterizing a self-enforcing agreement.

The paper is organized as follows. Section 2 presents the static game, Section 3 finds the static equilibrium, and Section 4 introduces the analysis of the repeated game. Section 5 concludes.

## 2. The model

Consider a buyer who awards a procurement project to one of two firms  $i = 1, 2$  by running a sealed-bid competitive tendering. The cost of each bidder is:

$$c_i = \theta_i + \psi(q_i)$$

The cost  $\theta$  is fixed and it does not change according to the provided quality. It also includes the cost each firm experiences in order to participate to the competitive tendering. We assume  $\theta_1 = \underline{\theta}, \theta_2 = \bar{\theta}$  with  $\bar{\theta} > \underline{\theta}$ , that is, firm 1 is the most efficient.  $\psi(q_i)$  is the variable cost of providing quality  $q_i$ . We follow Kim (1998) and assume that  $\psi(\cdot)$  is common to both firms.

The profit of each firm is:

$$\pi_i = p_i - \theta_i - \psi(q_i)$$

where  $p_i$  is the price paid by the buyer to firm  $i$  that delivers quality  $q_i$  at cost  $c_i$ . We also assume that the buyer requires fulfillment of a minimal quality standard denoted by  $\bar{q}$ . The level  $\bar{q}$  becomes the quality bid in the competitive tendering by both firms. Once awarded the competitive tendering the firm may shrink on quality and depart from  $\bar{q}$ . To represent this scenario, we define the effective quality as  $q_i = \bar{q} - m$ , with  $m = \{0, \bar{q}\}$ . The variable cost function respects the following conditions:  $\psi'(\cdot) \geq 0$ ,  $\psi''(\cdot) \geq 0$ ,  $\psi'(0) = 0$ ,  $\psi(0) = 0$ , in particular there exists some points along  $\psi(\cdot)$  with slope lower than one<sup>2</sup>. Also, we assume  $\bar{\theta} - \underline{\theta} > \psi(\bar{q})$ : firm 1 is much more efficient than firm 2. This assumption will be fundamental for the result of the paper, nevertheless it will not affect the quality of our results. The utility function of the buyer is as follows:

$$U = q_i - p_i$$

<sup>2</sup> The last assumption is purely technical, however its necessity will be clear in the proof of Lemma 1.

We assume that: *i)* the buyer perfectly observes the quality and the costs of the firms, and *ii)* the firms are fully informed. Assumption *i)* is in line with the common idea that a procurer is more informed on the cost of the firm than a standard regulator<sup>3</sup>. Although the buyer knows the costs of the firms she needs to run a competitive tendering to award the project. This apparently counterintuitive assumption actually fits many competitive tendering where the buyer knows *ex-ante* the efficiency of the bidders. This is the case of those procurement acquisitions repeated over time in which bidders are in general always the same and the buyer runs the competitive tendering only because mandatory by the law.

Let us introduce the three-stage static game  $G$  that will be the constituent game for the infinitely repeated framework introduced in Section 4. The timing of  $G$  is the following:

- **First stage** A reduced version of the sealed-bid competitive tendering in Burget and Che (2004) takes place. The buyer requires fulfillment of a minimal quality standard denoted by  $\bar{q}$ . When firms accept to take part to the competitive tendering they automatically commit to bid quality  $\bar{q}$ , therefore competitive bidding is only over price. Firms submit their bids simultaneously and noncooperatively. The highest score (or the lowest price) awards the competitive tendering. In the case of the same score the buyer uses a *tie-breaking rule* awarding the contract to the most efficient firm (firm 1)<sup>4</sup>.
- **Second stage** The contractor decides the effective level of quality and may depart from the required level.
- **Third stage** The buyer decides whether to handicap by an amount  $h$  the scoring rule of the opportunistic contractor in the next competitive tendering.

<sup>3</sup> As explained in Kim (1998), in case of public procurement the buyer is usually composed of managers coming from the private sector.

<sup>4</sup> A tie-breaking rule in case of equal score is also used in Kim (1998). However, he assumes that when bidders quote the same price the flip of coin determines the winner.

### 3. The equilibrium of the static game

We solve by backward induction. At the third stage the buyer simply decides the level of handicap,  $h > 0$ . Since handicapping will be effective from the next period, in this section we can only focus on the second and the first stage. We employ the technical assumption that when handicapping is applied it is assumed  $2\psi(\bar{q}) \leq h$ . This assumption does not affect the quality of the results<sup>5</sup>.

#### 3.1 Second stage: optimal effective quality

Once the competitive tendering has been awarded the contractor faces the following maximization problem:

$$\max_m p_i - \theta_i - \psi(\bar{q} - m)$$

solving w.r.t.  $m$  the solution is:

$$\psi'(\bar{q} - m) = 0$$

this means that  $m^* = \bar{q}$ . In the static game the contractor has an incentive not to deliver quality at all. The optimal quality will be  $q_i^* = 0$ . Since the static game ends at the third stage each contractor will behave opportunistically regardless the handicap.

#### 3.2 First stage: competitive tendering

Given the fixed level of quality required by the contract, when firm  $i$  is not handicapped it bids under the following scoring rule:

$$S_i = \bar{q} - p_i$$

On the other hand, when it is handicapped its scoring rule is:

$$S_i = \bar{q} - p_i - h$$

<sup>5</sup> As will be clarified in the Section 4, this is a necessary condition for the existence of a "cooperative" equilibrium in the dynamic game.

Since quality is not-contractible (because not-verifiable) the strategy of handicapping is fully discretionary and it does not need to be enforced by a court: the buyer may punish as she prefers right because a court can not do that<sup>6</sup>.

Following Burguet and Che (2004) we define the bidding advantage of firm 1 over firm 2 as:

$$\Delta = q_1 - \underline{\theta} - \psi(q_1) - [q_2 - \bar{\theta} - \psi(q_2)]$$

Since we are assuming the fulfillment of a minimal quality standard  $\bar{q}$  and identical variable cost, then the bidding advantage of firm 1, when it is handicapped, becomes:

$$\Delta = \bar{\theta} - \underline{\theta} - h$$

On the other hand, since we allow either firm to be handicapped, the bidding advantage of firm 1, when firm 2 is handicapped, is:

$$\tilde{\Delta} = \bar{\theta} - \underline{\theta} + h$$

However, since we are solving by backward induction, in the first stage firms anticipate that the optimal (effective) quality delivered in the next stage will be 0, therefore we have  $\psi(0) = 0$ .

The equilibrium bids, when firm 1 is handicapped, are given in the following proposition:

**Proposition 1** Given  $\Delta = \bar{\theta} - \underline{\theta} - h$ , the equilibrium bids of  $G$  are:

$$p_1^* = \underline{\theta} + \psi(0) + \max\{\Delta, 0\} = \underline{\theta} + \max\{\Delta, 0\}$$

$$p_2^* = \bar{\theta} + \psi(0) + \max\{-\Delta, 0\} = \bar{\theta} + \max\{-\Delta, 0\}.$$

The profits of the bidders are  $\pi_1 = \max\{\Delta, 0\}$  and  $\pi_2 = \max\{-\Delta, 0\}$ .

<sup>6</sup> The only constraint the buyer needs to respect is that the punishment must be dynamically consistent that, in fact, is what we show in the following section.

Proposition 1 says that, when the handicap is lower than the bidding advantage of firm 1, the competitive tendering is still awarded to the efficient firm that bids a price equal to the fixed cost of firm 2 minus the handicap. In other words, when the score of the efficient firm is reduced by an exogenous amount, then firm 1 needs to reduce its price by the same amount (bid more aggressively) in order to recover the score lost and keep winning the tendering.

To find the equilibrium bids in the static context we simply consider no handicap ( $h = 0$ ). In this case firm 1 wins the competitive tendering and the equilibrium bids are:

$$p_1 = \bar{\theta}$$

$$p_2 = \bar{\theta}$$

In this equilibrium the efficient firm is able to outbid the rival gaining a profit equal to its cost advantage. On the other hand, the equilibrium bids, when firm 2 is handicapped, are given in Proposition 2.

**Proposition 2** Given  $\tilde{\Delta} = \bar{\theta} - \underline{\theta} + h$ , the equilibrium bids of  $G$  are:

$$p_1^* = \underline{\theta} + \psi(0) + \tilde{\Delta} = \bar{\theta} + h \quad (1)$$

$$p_2^* = \bar{\theta} + \psi(0) = \bar{\theta}$$

the profits of the bidders are  $\tilde{\pi}_1 = \tilde{\Delta}$  and  $\tilde{\pi}_2 = 0$ .

When firm 2 is handicapped the bidding advantage of firm 1 is higher than in the previous case. Handicapping the less efficient firm is equivalent to increase the score of the efficient one, then, in equilibrium, firm 1 increases its bid (with respect to the case of no handicap) by an amount equal to the handicap of the rival. In this equilibrium the efficient firm still wins the competitive tendering and also increases its profit.

## 4. The dynamic game

In this section we introduce the dynamic game as an infinite repetition of the static game  $G$ . Since  $t = 1$  on, the equilibrium of  $G$  depends on  $h$ , then in what follows we anticipate three possible equilibria of  $G$  according to  $h$ .

### 4.1 The role of handicapping

When firm 1 is handicapped and both firms bid the same level of quality, then its bidding advantage is  $\Delta = \bar{\theta} - \underline{\theta} - h$ . The difference in the fixed costs  $(\bar{\theta} - \underline{\theta})$  measures the asymmetry among competitors and it denotes the upper bound level of handicap that makes the efficient firm to be awarded the contract. In other words, the level of  $h$  can be also seen as an increase in the fixed cost of firm 1. In particular, when  $h = \bar{\theta} - \underline{\theta}$ , handicapping firm 1 makes both competitors alike and the bidding advantage is exactly compensated. A level  $h < \bar{\theta} - \underline{\theta}$  makes firm 1 still more efficient, whereas  $h > \bar{\theta} - \underline{\theta}$  replicates the scenario in which firm 1 has a higher fixed cost than firm 2. Let us consider the following cases  $A$ ,  $B$  and  $C$ .

A)  $h < \bar{\theta} - \underline{\theta}$ : firm 1 wins the competitive tendering and the equilibrium is:

$$p_1^A = \bar{\theta} - h$$

$$p_2^A = \bar{\theta}$$

$$\pi_1^A = \bar{\theta} - \underline{\theta} - h$$

$$\pi_2^A = 0$$

In this scenario the handicap is not harsh enough to switch contractors. In particular, when firm 1 is handicapped by  $h$ , then the bid  $p_1^A = \bar{\theta} - h$  would make the efficient firm outbid by firm 2.

B)  $h = \bar{\theta} - \underline{\theta}$ : nobody wins the competitive tendering, however the most efficient firm (firm 1) is awarded the contract by the *tie-breaking rule*. The equilibrium is:

$$p_1^B = \underline{\theta}$$

$$p_2^B = \bar{\theta}$$

$$\pi_1^B = 0$$

$$\pi_2^B = 0$$

Note that in this scenario the handicap makes the bidders symmetric.

c)  $h > \bar{\theta} - \underline{\theta}$  : firm 2 wins the competitive tendering and the equilibrium is:

$$p_1^C = \underline{\theta}$$

$$p_2^C = \underline{\theta} + h$$

$$\pi_1^C = 0$$

$$\pi_2^C = h - \bar{\theta} + \underline{\theta}$$

This higher level of handicap induces to switch contractors. The efficient firm is no longer able to outbid the less efficient one that wins the competitive tendering by bidding less aggressively than in *A*) and *B*)<sup>7</sup>. In the next section we will use scenario C as benchmark to study the trade-off from handicapping: although a sufficiently high handicap may give incentive not to shrink, it may implicitly award the contract to the less efficient firm that wins the next competitive tendering by bidding less aggressively.

The following Corollary defines the equilibrium bids of the stage game when the contractor decides to deliver the quality  $\bar{q}$  and no handicapping is applied.

**Corollary 1** *In the stage game, when no handicap is applied, firm 1 wins the competitive tendering even though it will deliver  $\bar{q}$  at the last stage. The equilibrium bids remain  $p_1^* = p_1^* = \bar{\theta}$ .*

Corollary 1 shows that even though firm 1 decided to fulfill the quality requirement it would win the tendering by still bidding  $p_1^* = \bar{\theta}$ . We will use this result to define the profit gained by the contractor in the dynamic game when no opportunistic behavior arises.

<sup>7</sup> The winning bids in C is such that  $p_2^C = \underline{\theta} + h > p_1^A = \bar{\theta} - h > p_1^B = \underline{\theta}$ .

## 4.2 The repeated game

Let  $G^\infty$  be the supergame obtained by an infinite repetition of the game  $G$ . We assume that  $\underline{\theta}$  and  $\bar{\theta}$  are fixed over time. Let  $\delta$  be the discount factor common to the firms and the buyer. Let  $H_t$  be the common knowledge vector of previous actions undertaken by the players in periods up to  $t-1$ . Also, let  $H_0$  be the history at time 0. Consider now the following specifications of the history given in the following definitions.

**Definition 1** Let  $\hat{H}_t$  be the history at time  $t$  such that up to the second stage of time  $t$  the contractor produces  $\bar{q}$  and no handicap has occurred.

**Definition 2** Let  $\tilde{H}_t$  be the history at time  $t$  such that up to time  $t-1$  the contractor produces  $\bar{q}$  and no handicap has occurred.

Given the history in Definitions 1-2, in the Definition 3-5 we anticipate the "stick and carrot" strategies pioneered in Abreu (1986) that will characterize a SNE of  $G^\infty$ .

**Definition 3** Let  $s_t^b$  be the strategy of the buyer at time  $t$  such that:

- If  $H_t = H_0$ , no handicap is applied.
- If  $H_t = \hat{H}_t$ , no handicap is applied.
- otherwise she decides to handicap ( $h$ ) the cheating contractor for one period, after which revert to no handicap.

**Definition 4** Let  $s_t^1$  be the strategy of firm 1 at time  $t$  such that:

- If  $H_t = H_0$ , it deliver  $\bar{q}$
- If  $H_t = \tilde{H}_t$ , once the competitive tendering has run, it delivers  $\bar{q}$
- If the buyer deviates from its strategy and firm 1 is handicapped even though it delivers  $\bar{q}$ , then firm 1 decides to deliver  $q^*$  for one period, after which revert to  $\bar{q}$ .

**Definition 5** Let  $s_t^2$  be the strategy of firm 2 at time  $t$  such that in every period (included  $t=0$ ) it delivers  $q^*$ .

In this setting the presence of the less efficient firm serves as threat for the most efficient one who in general would win the competitive tendering and deliver the service.

Given Corollary 1, the static profit for firm 1 when  $s_i^1$ ,  $s_i^b$  and  $s_i^2$  are respected (firm 1 wins and it does not shrink and the buyer does not handicap) is:

$$\bar{\pi}_1 = \bar{\theta} - \underline{\theta} - \psi(\bar{q})$$

That is, firm 1 wins the competitive tendering by bidding  $p_1 = \bar{\theta}$  and providing  $\bar{q}$ . Firm 1 still gains positive profit even bidding a price equal to the fixed cost of firm 2 and providing quality  $\bar{q}$ .

Its discounted payoff is:

$$\bar{V}_1 = \frac{1}{1-\delta} \bar{\pi}_1$$

The discounted profit for the buyer is:

$$\bar{U} = \frac{1}{1-\delta} (\bar{q} - \bar{\theta})$$

When firm 1 respects the quality  $\bar{q}$ , the buyer gains exactly  $\bar{q}$  and rewards the contractor with a payment equal to the bid price ( $\bar{\theta}$ ).

In line with the "Folk theorem" the enforcement of strategies  $s_i^1$ ,  $s_i^b$  and  $s_i^2$  depends on  $\delta$  and, more interestingly, on  $h$ . Thus let us consider the following cases:

A)  $h \leq \bar{\theta} - \underline{\theta}$ . Let  $h_A$  be a level of handicap at most equal to  $\bar{\theta} - \underline{\theta}$ . In this scenario firm 1 still wins the competitive tendering and the static equilibrium bids and profits are:

$$p_1^A = \bar{\theta} - h_A$$

$$p_2^A = \bar{\theta}$$

(2)

$$\pi_1^A = \bar{\theta} - \underline{\theta} - h_A$$

$$\pi_2^A = 0$$

<sup>8</sup> We recall that when  $h_A = \bar{\theta} - \underline{\theta}$  firm 1 wins the competitive tendering by the *tie-breaking rule*.

Now in the following Lemma we introduce a necessary condition for a SNE to exist. According to Abreu (1986) the necessary conditions for the strategies  $s_t^1$ ,  $s_t^b$  and  $s_t^2$  to characterize a SNE are: i) the contractor weakly prefers to respect the minimum quality requirement and the buyer never handicaps (incentive compatibility constraint), and ii) the punishment strategy is credible: once the game ends up in the punishment phase then the players effectively acts as explained in  $s_t^1$ ,  $s_t^b$  and  $s_t^2$ .

**Lemma 1** When  $h \leq \bar{\theta} - \underline{\theta}$ , the necessary conditions for  $s_t^1$ ,  $s_t^b$  and  $s_t^2$  to characterize a SNE of  $G^\infty$

$$\text{are } \frac{\Psi(\bar{q})}{h_A - \Psi(\bar{q})} \equiv \tilde{\delta}_A \leq \delta \text{ and } \bar{q} \geq h.$$

Although handicapping is such that the most efficient firm keeps winning the competitive tendering, cheating is not a so optimum strategy as it seems. There are two effects working at this level: cheating on  $q$  will directly increase the utility of the contractor, nevertheless this handicap will induce the efficient firm to bid a lower price in order to win the next competitive tenderings. When the variable cost of producing  $q$  is sufficiently low then the gain from cheating on  $q$  is also low. In this case, firm 1 prefers not to cheat at  $t = 0$  by gaining the "cooperative" profit over time rather than cheating and winning the next competitive tenderings at a lower price. The only condition for the buyer to respect its strategy is that the handicap is at most equal to the required quality, regardless to the discount factor. Given the bid in (2), the handicap represents the gain for the buyer in terms of more aggressive bidding: when firm 1 does not deliver quality it is handicapped and its equilibrium bid is decreasing in  $h$ . Hence, when the gain from respecting the agreement ( $\bar{q}$ ) is at least equal to the gain from handicapping ( $h$ ), then the buyer never deviates from its strategy.

B)  $h > \bar{\theta} - \underline{\theta}$ . Let  $h_B$  be a level of handicap strictly higher than  $\bar{\theta} - \underline{\theta}$ . In this scenario firm 2 wins the competitive tendering and the static equilibrium is:

$$p_1^B = \underline{\theta}$$

$$p_2^B = \underline{\theta} + h_B$$

(3)

$$\pi_1^B = 0$$

$$\pi_2^B = h_B - \bar{\theta} + \underline{\theta}$$

**Lemma 2** When  $h > \bar{\theta} - \underline{\theta}$ , the efficient firm never respects  $s_t^1$  and finds it optimal to deliver  $q^*$ , then  $s_t^1$ ,  $s_t^b$  and  $s_t^2$  cannot characterize a SNE of  $G^\infty$ .

Lemma 2 says that when the handicap is higher than the bidding advantage of firm 1, the efficient firm never respects its strategy  $s_t^1$ . This result is due to the behavior of the less efficient firm, that is, firm 2 finds it optimal not to deliver quality in every period. Accordingly firm 1 prefers to provide zero quality, lose the tendering for one period and be awarded the contract when firm 2 is handicapped rather than delivering  $\bar{q}$  for ever.

Since a too harsh handicap does not induce the most efficient firm to deliver  $\bar{q}$ , the strategies in definition 1-3 will never characterize a SNE under  $h_B$ . Thus the only handicap we can consider is  $h_A$ . However, to show that the handicap  $h_A$  is a SNE we need to check for the credibility of  $s_t^b$ . The following Proposition shows that the credible level of handicap is  $h = \bar{\theta} - \underline{\theta}$ .

**Proposition 3** When  $h = \bar{\theta} - \underline{\theta}$  and  $\tilde{\delta}_A \leq \delta$ ,  $s_i^b$ ,  $s_i^1$  and  $s_i^2$  characterize a SNE of  $G^\infty$  in which, in every period, the efficient firm is awarded the contract and delivers  $\bar{q}$ .

Proposition 3 highlights the trade off from handicapping by showing that a strong handicap, as deterrence for moral hazard on quality, does not benefit the buyer when the contract is awarded by a competitive tendering. In particular, the best strategy for the buyer is to punish the cheating efficient firm by choosing a level of handicap that makes the heterogeneous competitors more symmetric. A handicap higher than the bidding advantage of the efficient firm makes the buyer worse off because of two effects. First, the less efficient supplier wins the next competitive tendering by bidding less aggressively and providing zero quality. Second, this behavior induces the efficient firm to behave opportunistically: it prefers to lose the tendering for one period but be awarded the contract at less aggressive conditions when firm 2 is handicapped.

Proposition 3 shows that the optimal handicapping strategy also depends on the degree of asymmetry among competitors. Consider the handicap  $h = \bar{\theta} - \underline{\theta}$ . It is straightforward to see that the efficient contractor's willingness to deliver  $\bar{q}$  is increasing in  $(\bar{\theta} - \underline{\theta})^9$ . When competitors are very asymmetric the buyer needs a harsh handicap to make the threat of switching contractor more effective and induce the efficient firm to deliver  $\bar{q}$ <sup>10</sup>.

<sup>9</sup> We recall that, under  $h_A = \bar{\theta} - \underline{\theta}$ , the condition for the efficient firm to deliver  $\bar{q}$  is  $\frac{\psi(\bar{q})}{h_A - \psi(\bar{q})} \leq \delta$ . Given  $\delta \in [0, 1]$ , the willingness to respect  $\bar{q}$  is  $\left(1 - \frac{\psi(\bar{q})}{\bar{\theta} - \underline{\theta} - \psi(\bar{q})}\right)$ , that is increasing in  $(\bar{\theta} - \underline{\theta})$ .

<sup>10</sup> In the extreme case of symmetry ( $\bar{\theta} - \underline{\theta} = 0$ ) the threat of switching contractor by making competitors more alike does not work and the efficient firm behaves opportunistically. However, by assuming  $\bar{\theta} > \underline{\theta} + \psi(\bar{q})$ , we rule out the case of perfect symmetry because, when  $\bar{\theta} - \underline{\theta} = 0$ , the profit of the contractor would be  $\pi_1 < 0$ .

## 5. Conclusions

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This paper provides a solution to deter ex post moral hazard in repeated procurement when the quality delivered by the contractor is not verifiable by a third part. We have considered a framework in which a long-run relationship between a buyer and an efficient seller is built on a series of short-run contracts. In principle, the presence of a less efficient supplier puts an upper bound to the incumbent seller's per-period profit. However, the efficient seller may be tempted to increase its profit by not delivering the agreed level of (unverifiable) quality.

We have then explored how the buyer would optimally use a discipline device that consists in altering at her discretion the incumbent seller's score in subsequent competitive tendering (handicapping). In other words, what would happen if the buyer could resort to an indirect punishment device that goes through the modification of the "playing field" between the two competitors? Our answer is that extreme forms of punishment are never credible, that is, it is never in the buyer's interest to kick the deviant incumbent out of the playing field. The buyer's optimal strategy is, rather, to perfectly level the playing field once if the incumbent had deviated from the cooperative strategy (i.e., deliver the agreed level of quality).

There are at least two directions for further investigation. First, we have implicitly assumed that both the buyer and the incumbent contractor observe a perfectly correlated signal about delivered quality. It would be worth testing the robustness of our predictions when the two signals are imperfectly correlated as in McLeod (2003). Secondly, the assumption of complete information about firms' efficiency levels is instrumental to the buyer for fine-tuning the optimal handicapping strategy. When the buyer is uncertain about firms' costs the former has to rely on equilibrium bids to learn about firms' efficiency levels. The interaction between learning and handicapping certainly deserves closer attention.

## 6. Appendix

### *Proof of Proposition 1*

See B&C (2004). The difference with their paper is that our bidding advantage collapses to  $\Delta = \bar{\theta} - \underline{\theta} - h$  because we assume a fixed  $\bar{q}$  instead of a continuous quality. Also, differently from B&C, in our model quality and price are chosen sequentially and not simultaneously, therefore by backward induction we have  $q = 0$  in the equilibrium price.

### *Proof of Corollary 1*

Proposition 1 shows that firm 1 always wins the competitive tendering when both firms at the first stage make their bids anticipating that they will not deliver quality at the second stage. Hence, to prove Corollary 1 it remains to show that firm 1 wins the competitive tendering even when it anticipates to deliver  $\bar{q}$ . Consider that the most aggressive bid by firm 2 is  $p_2 = \bar{\theta}$ , that is the price bid when firm 2 anticipates that it will not deliver quality.

When firm 1 wants to deliver quality  $\bar{q}$ , since  $\bar{\theta} - \underline{\theta} > \psi(\bar{q})$ , it may win the competitive tendering and gain a positive profit with all the bids from  $\underline{\theta} + \psi(\bar{q})$  to  $\bar{\theta}$ . Thus, it is possible to see that the only equilibrium when firm 1 decides to deliver  $\bar{q}$  is  $p_{1,2} = \bar{\theta}$ .

The proof of Proposition 2 follows the proof of Proposition 1, then we omit it.

### *Proof of Lemma 1.*

Since, given  $h_A$ , firm 2 always bids  $p_2^A$  and never wins the competitive tendering, we can consider only the strategy of firm 1 and the buyer. We consider the repeated game starting at  $t = 0$  and sketch the proof over two points. 1) Firstly, consider firm 1. When firm 1 cheats its discounted profit is:

$$V_1^A = \pi_1 + \delta\pi_1^A + \sum_{t=2}^{\infty} \delta^t \bar{\pi}_1$$

At time  $t = 0$ , if firm 1 cheats and produces  $q^* = 0$ , it gains  $\pi_1$ . At  $t = 1$ , according to  $s_t^b$ , the handicap is applied. In this case, firm 1 still wins the competitive tendering but with a more aggressive price, then it gains  $\pi_1^A$ . Since  $t = 2$  on, no handicap is applied and firm 1 reverts to  $\bar{q}$  by gaining  $\bar{\pi}_1$ . Hence, the condition for firm 1 not to cheat on quality is:

$$\bar{V}_1 \geq V_1^A$$

that holds when:

$$\frac{\psi(\bar{q})}{h_A - \psi(\bar{q})} \equiv \tilde{\delta}_A \leq \delta$$

To characterize a SNE we also need to show that the punishment strategy of firm 1 is credible. This means that firm 1 should not deviate from his strategy once the punishment phase gets started. The punishment defined in  $s_i^1$  (that is, firm 1 delivers  $q^* = 0$  only for one period and then reverts to  $\bar{q}$ ) is credible when firm 1 has no incentive to deviate from  $q^* = 0$  during the period of the punishment. Nevertheless, since  $q^* = 0$  is its best reply in the static game, then during the punishment firm 1 does not deviate from  $q^* = 0$ . Hence,  $s_i^1$  is credible and the necessary condition for  $s_i^1$  to characterize a SNE is  $\tilde{\delta}_A \leq \delta$ . Moreover, it is possible to see that without the assumption  $2\psi(\bar{q}) < h_A$  we have  $\tilde{\delta}_A > 1$ . This implies that, in this case, strategy  $s_i^1$  cannot characterize a SNE because firm 1 never delivers  $\bar{q}$ <sup>11</sup>. 2) Second, consider the buyer. When the punishment, as defined in  $s_i^b$  and  $s_i^1$ , starts, then the effective quality is zero for one period and then it reverts to  $\bar{q}$ . The discounted utility of the buyer if she deviates is:

$$U_A = (\bar{q} - \bar{\theta}) + \delta(0 - \bar{\theta} + h_A) + \frac{\delta^2}{1 - \delta}(\bar{q} - \bar{\theta})$$

If at time  $t = 0$  the buyer deviates and decide to handicap firm 1 even though it has delivered  $\bar{q}$ , she receive  $\bar{q}$ , pays  $\bar{\theta}$  and her profit is  $(\bar{q} - \bar{\theta})$ . At time  $t = 1$ , under  $h_A$ , firm 1 wins the competitive tendering at price  $p_1^A = \bar{\theta} - h_A$  and, according to  $s_i^1$ , it delivers zero quality. Since  $t = 2$  on, the buyer and the efficient firm revert to their strategy. In particular, firm 1 reverts to  $\bar{q}$  and wins all the competitive tenderings at price  $p_1 = \bar{\theta}$  and no handicap is applied. In this scenario the buyer gains  $(\bar{q} - \bar{\theta})$  in every period. Thus the necessary condition for the buyer not to cheat is:

$$\bar{U} \geq U_A \quad (44)$$

that holds for every  $\delta \in [0, 1]$  when  $\bar{q} \geq h_A$ <sup>12</sup>.

<sup>11</sup> Since we show that even under  $h > \bar{\theta} - \underline{\theta}$ , the strategy  $s_i^1$  will not characterize a SNE as well, then without the assumption  $2\psi(\bar{q}) < h_A$  our equilibrium strategies collapses.

<sup>12</sup> Since we assume  $h_A \geq 2\psi(\bar{q})$  and  $\bar{\theta} - \underline{\theta} > \psi(\bar{q})$ , the necessary condition for  $\bar{q} > h_A$  to hold is  $\bar{q} > 2\psi(\bar{q})$ . However, whether the condition  $\bar{q} \geq h_A$  holds or not depends on the slope of the variable cost. In particular, since  $\psi(\bar{q})$  is convex, the choice of a level  $\bar{q}$  such that  $\bar{q} > 2\psi(\bar{q})$  depends on the slope and the degree of convexity of  $\psi(\cdot)$ . It is straightforward to show that, whenever the slope of  $\psi(\cdot)$  is lower than one, there always exists a level of  $\bar{q}$  such that  $\bar{q} > 2\psi(\bar{q})$  (then  $\bar{q} > h_A$  holds). On the other hand, when the slope of  $\psi(\cdot)$  is always higher than one, we always have  $\bar{q} < 2\psi(\bar{q})$ , then  $\bar{q} < h_A$ . In particular, if we assume a linear variable cost  $\psi \bar{q}$  with slope  $\psi < \frac{1}{2}$ , then the necessary condition for  $\bar{q} > h_A$  to hold is always respected.

*Proof of Lemma 2*

First, we show that the strategy of firm 1,  $s_t^1$ , does not characterize a SNE. Since the enforcing of the strategy  $s_t^1$  is a necessary condition for strategies  $s_t^1$ ,  $s_t^b$  and  $s_t^2$  to characterize a SNE of  $G^\infty$ , then we can avoid the analysis of strategy  $s_t^b$ . We consider the repeated game starting at  $t = 0$ . The proof proceeds over two steps. 1) Firstly, we show that in every period of the repeated game firm 2 always delivers  $q^*$  (it never delivers a positive quality). Assume that firm 2 delivers  $\bar{q}$ ; in this case it gains  $\pi_2 = h_B - \bar{\theta} + \underline{\theta} - \psi(\bar{q})$ ; however, according to the strategy  $s_t^b$ , no handicap will be applied, then firm 2 will lose the next competitive tendering. On the other hand, if firm 2 delivered  $q = 0$  it would gain  $\pi_2^B$  as in (3); in this case firm 2 will be handicapped and it will lose the next competitive tendering as well. Hence, given  $\pi_2 < \pi_2^B$ , firm 2 always cheats on quality under  $h_B$ . 2) Secondly, consider firm 1. We show that firm 1 always prefers to cheat on quality (deliver  $q^* = 0$ ) instead of delivering  $\bar{q}$ . Given the result in point 1, by Proposition 2 the bidding advantage of firm 1 becomes  $\tilde{\Delta} = \bar{\theta} - \underline{\theta} + h_B$ . Thus, the equilibrium bids in (1) imply that firm 1 wins the competitive tendering by bidding  $p_1^* = \bar{\theta} + h_B$ .

Let  $\tilde{\pi}_{1,q}$  denote the profit firm 1 gains by bidding  $p_1^* = \bar{\theta} + h_B$  and providing  $\bar{q}$ , with:

$$\tilde{\pi}_{1,q} = h_B + \bar{\theta} - \underline{\theta} - \psi(\bar{q})$$

Deviation entails that firm 1 produces  $q^*$  ( $q = 0$ ) for one period and then revert to  $\bar{q}$ . Hence, when firm 1 deviates its discounted profit is:

$$V_1^B = \pi_1 + \delta(0) + \delta^2 \tilde{\pi}_{1,q} + \sum_{t=3}^{\infty} \delta^t \bar{\pi}_1$$

If firm 1 deviates at  $t = 0$  its current profit is  $\pi_1$ . At  $t = 1$  it will be handicapped and the competitive tendering will be won by firm 2, then firm 1 gains zero. By point 1, we know that firm 2 always delivers  $q^* = 0$ , then it will be handicapped as well at time  $t = 1$ <sup>13</sup>. At time  $t = 2$ , firm 1 wins again the competitive tenderings at price  $p_1^* = \bar{\theta} + h_B$ . Since  $t = 2$  on firm 1 reverts to  $\bar{q}$ , then it gains  $\tilde{\pi}_{1,q}$  at  $t = 2$  and  $\bar{\pi}_1$  since  $t = 3$  on. The condition for firm 1 to always deliver  $\bar{q}$  is:

$$\bar{V}_1 \geq V_1^B$$

that never holds for every  $\delta \in [0, 1]$ <sup>14</sup>.

<sup>13</sup> We recall that handicapping at time  $t = 1$  entails that the reduction in the score will be applied on the scoring rule at time  $t = 2$ .

<sup>14</sup> By rewriting the inequality above we have  $\frac{1}{1-\delta} \bar{\pi}_1 \geq \pi_1 + \delta^2 \tilde{\pi}_{1,q} + \sum_{t=3}^{\infty} \delta^t \bar{\pi}_1$ , that becomes  $\delta^2 h - \delta(\bar{\theta} - \underline{\theta} - \psi) + \psi \leq 0$ . This

$$\text{holds for every } \frac{(\bar{\theta} - \underline{\theta} - \psi) - \sqrt{(\bar{\theta} - \underline{\theta} - \psi)^2 - 4/h\psi}}{2h} \leq \delta \leq \frac{(\bar{\theta} - \underline{\theta} - \psi) + \sqrt{(\bar{\theta} - \underline{\theta} - \psi)^2 - 4/h\psi}}{2h}.$$

$$\text{However, it is straightforward to show that } \frac{(\bar{\theta} - \underline{\theta} - \psi) + \sqrt{(\bar{\theta} - \underline{\theta} - \psi)^2 - 4/h\psi}}{2h} < 0.$$

Furthermore, the threat of firm 1 is credible because in the period of punishment it delivers  $q^* = 0$ , that is its short-run best reaction.

*Proof of Proposition 3*

*Proof.* Given the results in Lemma 1-2, the last step to characterize a SNE is showing which level of  $h_A$  is a credible punishment for the buyer. The punishment is credible when, given the choice of the contractor, the action played by the buyer really represents her best reply. Hence, the credible punishment is the level of  $h$  allowing the buyer the highest utility given that the contractor has cheated. Since, by Lemma 2,  $s_i^b$ ,  $s_i^1$  and  $s_i^2$  cannot characterize a SNE under  $h_B$ , in order to find the credible handicap we focus only on  $h_A$ . Under  $h_A$ , once the punishment gets started, the buyer gains  $U_A = (0 - \bar{\theta} + h_A) + \frac{\delta}{1-\delta} (\bar{q} - \bar{\theta})$ , that is maximized when  $h_A = \bar{\theta} - \underline{\theta}$ . Hence,  $h_A = \bar{\theta} - \underline{\theta}$  is the credible strategy.

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