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Preliminary and incomplete version

Abstract

Governments as well as private firms face the risk that a contractor goes bankrupt before or during the completion of the project. We investigate in such an environment the bidding behavior in procurement auctions. Among other results we show: i) The revenue equivalence result breaks down. ii) The question, whether a second price sealed bid auction fares better or worse than a first price sealed bid auction depends not only on the preferences of the procuring agency, but also on the distribution of the cost uncertainty of the individual firms. Thus a general ranking is not possible. iii) In contrast to standard auction theory, the following mechanisms might fare better than a standard auction: 1) split awards and 2) rationing.

JEL-Classification: D44

Keywords: Auctions, Revenue Equivalence, Bankruptcy, Risk Management

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1 Introduction

We examine the impact of ex-post bankruptcy in procurement auctions on the bidding behavior and on the revenue and efficiency of such auctions. While the main argument for the use of procurement auctions is the low price generated by competition in the bidding process, most theoretical models ignore the fact that contractors face the risk of bankruptcy before the completion of the project. Especially in an environment with uncertainty about the future costs. The focus of this paper is that procurement mechanisms can enhance the risk of bankruptcy in an uncertain environment through too aggressive bidding. A bankrupt contractor may then cause the procurement agency costs of additional auctions, delays and even the risk of non-completion. Bankruptcy for instance affected the costs of the 2004 olympic games, where the costs of the marathon course increased dramatically, after the main contractor declared bankruptcy. In short, there is a trade-off between low prices generated by the procurement mechanisms and the risk of awarding the contract to conditions that lead to bankruptcy, induced by the bidding incentives of the mechanism.[1]

Procurement auctions (also called tenders) are a common instrument by private and also by public entities to buy goods or services from suppliers. In a procurement auction the buyer (procurement agency) has a valuation or maximal willingness to pay for the contract and the sellers (bidders) have costs of producing the good or service. Many contracts are awarded the following way: The procurement agency announces the specifications of the contract, the contractors give the agency a sealed envelope with their offers and the contract is awarded to the company with the lowest price. Article 30 PWD (Public Works Directive) of the European Union says for instance" 1. The criteria on which the contracting authorities shall base the award of contracts shall be: (a) ... *the lowest price only*;...". So without explicitly calling this an auction, the rules are the same as in the first price sealed bid auction, where the bidder with the lowest bid wins the contract and receives his own bid as the price (and the loosing bidders receive nothing). In 2002 the total volume of public procurement in the EU – i.e. the purchases of goods, services and public works only by governments and public entities - is estimated at about

[1]Especially in industries like the construction industry which has high default rates because everything can be subcontracted. This led to mechanisms like surety bonds and default insurance, (Parlane (2003), Calveras et al. (2003) for regulatory issues).

16% of the Union's GDP or 1500 billion.[2] Apart from public procurement there are many examples in the private sector of the successful implementation of procurement auctions (e.g. B2B Marketplaces), so a better understanding what drives the bidding behavior in procurement auctions is needed.

Despite the advantages of auctions (10%-20% cheaper, fast and cheap mechanism) there are also risks. Probably most important is the risk of a cost overrun of the winning contractor, which means that the realized costs are higher than anticipated and are not covered by the payment.[3] If that is the case some companies might want to declare bankruptcy to avoid losses. Every contract in an uncertain environment incorporates the risk of a cost overrun and the resulting risk of non-completion, caused by a possible bankruptcy of the contractor. A recent report of the European Commission accentuates that "clients often underestimate or pay inadequate attention to the risks of abnormally low bids, especially the possibilities of bankruptcy and failure of enterprises, both principal and subcontractors, during execution of the work."[4] The reasons why costs can be higher than anticipated and therefore higher than the price are manifold. First, human issues like simply erroneous calculation by the bidder or too optimistic bidding, which reflect irrational behavior (or imperfect information?). Second, environmental issues like the uncertainty of the project, demand shocks in the future etc. And third and most important, characteristics of the contractors like financial need of the company, if the auction is the only project the company operates, or financial needs out of other projects that have to be covered.[5] In a recent article Arditi et al. (2000) investigate the factors associated with company failures in the US construction industry, which is an industry with high uncertainty about future states of nature (mainly about the costs). The weighted occurrence of human issues like lack of knowledge explains 7.5% of company failures. Budgetary issues like heavy operating losses and insufficient profit explain 60.2% of failures. And industry weakness is also a major factor with 22.7% of failures, which is an important issue because industry weakness increases competition (due to over-capacities)

[2] A recent change in European Law (Directive 2004/18/EC) will allow public entities to run electronic auctions, so auctions will become an even more important feature of public procurement.

[3] the price payed by the agency is the payment received by he winning bidder. When a winning bid is very low in relation to the other bids, it is more likely that there will be a cost overrun and therefore those bids are called abnormally low.

[4] Also the stylized facts lead to the conclusion that bankrupt contractors are an important topic. Alone in Germany more than 40.000 firms went bankrupt in the year 2003.

[5] The logic of the bidding behavior in this case is similar to the well known 'gambling for resurrection' in the finance literature, where individuals behave more risk loving, when the leverage is high.

and therefore reduces profits and leads to bankruptcy via abnormally low bids. So the first relevant question is, will the bidding behavior in a competitive environment like a procurement auction be affected by the possibility to declare bankruptcy? In more detail, what is the bidding behavior in the standard auctions if bidders can go bankrupt and how high is the risk of bankruptcy in that auction formats?[6] And second, which mechanisms fare better than the standard auctions, taking the goals of the auctioneers into account? The results also hold for selling auctions.

In the first part of our analysis we characterize the equilibrium bidding strategies and the expected prices and revenues for the first price sealed bid (FPSB-auction) and the second price sealed bid auction (SPSB-auction). We also extend our analysis to reserve prices and entry fees and allow for common values and asymmetries. In the second part, we examine different auction formats that fare better and we can give additional explanation to the existing literature why rationing or split award auctions might be preferred to standard auctions. We study the bidding behavior in an environment with independently and identically distributed private costs. And introduce additional cost-uncertainty via a discrete error term Δ over the costs distribution, which is common knowledge to all bidders. So the realization of the private costs for each bidder can either be c (low) or $c + \Delta$ (high) with some probability and can reflect all the reasons for bankruptcy mentioned above. The relevant regions are shown in Figure 1, where the shaded area between c and $c + \Delta$ indicates bankruptcy. If a bid is above the shaded area, the bidder will never go bankrupt.

[6] A standard auction is a auction where the contract is awarded to the lowest type and the highest type gets no surplus (never wins).

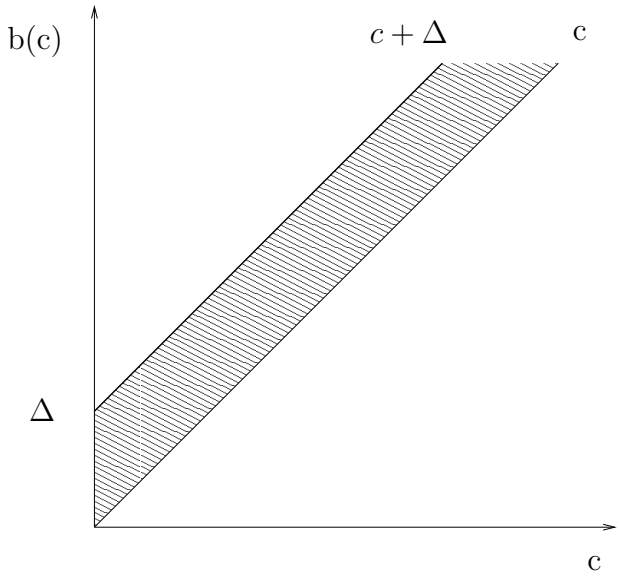

Figure 1: Bankruptcy

The utility of a bidder not winning the auction is zero and $-\epsilon$ if the winning bidder goes bankrupt. The intuition for the bidding behavior in an auction with ex-post bankruptcy is the following: Bidders have uncertainty about their true (ex-post) costs and each bidder can declare bankruptcy if the realized costs are higher than the payment ($\equiv$ cost overrun).[7] Which only happens if the costs are high ($c + \Delta$). So the removal of possible negative payoffs in the presence of uncertainty changes the price into a (long) put option in the case of a cost overrun. Figure 2 shows the profit of bidder i with cost c_i, for the distribution of her bids. So when the costs are $c_i + \Delta$ and the bid is smaller, than the bidder declares bankruptcy and has a payoff of -ϵ.

[7]We assume that the bidders have no budgets and that the game is a one shot game. Introducing budgets would be the same analysis but with a weaker impact of a cost overrun. Repeated games would be more realistic but are not solvable.

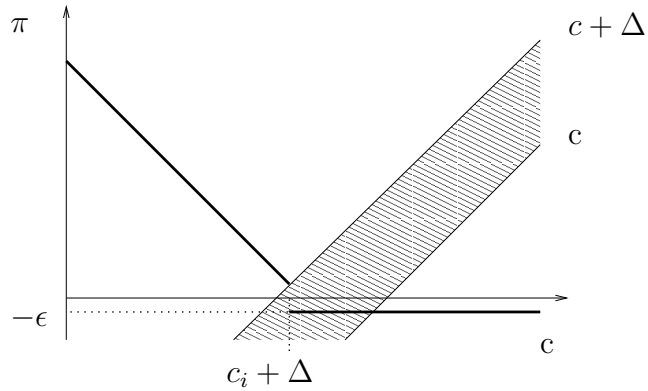

Figure 2: Profit function

The (weakly) dominant bidding strategy in a SPSB-auction is to bid c.[8] The price in the SPSB-auction is determined by the second lowest bid and therefore each bidder will bid as low as she can. The probability of bankruptcy is determined if the payment is below or above the threshold and that depends on the distribution of the second lowest bid and is therefore random. The bidding strategy in a FPSB-auction depends on the number of bidders. Assume for the moment that all bidders follow a bidding strategy that lies always above the threshold $c+\Delta$. In that situation bidders with very high costs nearly never win. But they can bid below $c + \Delta$, which raises their probability of winning and increases their payoff (first order effect) at the cost that they have to declare bankruptcy if the realization of the cost is high. The gain in the probability of winning is larger than the loss, because the loss is limited to ϵ (second order effect). So the error term works like a discount on the bidders own cost. If competition is fierce (many bidders (n large)) the aggressive bidding behavior of the high cost bidders forces also bidders with lower costs to respond with more aggressive bidding, and so on. In equilibrium each bidder has an incentive to bid below $c+\Delta$ and this leads to a probability of ex-post bankruptcy as high as Prob$[c+\Delta]$ which is certain. If there are only few bidders a special case arises and the equilibrium bid function will have a jump. While high cost bidders still have an incentive to shade their bid below $c + \Delta$ to win more often, low cost bidders can abstain from bidding aggressive. Shading their bid below $c + \Delta$ would raise the probability of winning for a low cost bidder only slightly, but it would reduce the profit substantially. Therefore low cost bidders will not bid below the threshold. So the first result of our paper is that

[8]For simplicity we assume here that $\epsilon \to 0$.

the expected price in both auction formats (for n large) is the same, the probability of bankruptcy and the expected payment differs and revenue equivalence breaks down.

In addition to that we investigate extensions like reserve prices and entry fees, give examples for different environments with common values and asymmetries and derive results for procurement mechanisms like rationing, lotteries, multi-sourcing and other methods. All those methods are used quite often and therefore we give here some examples. Using a reserve price or entry fees in a standard auction can increase the agency's utility by excluding bidders with high costs and therefore lowering the expected price. But there is also an efficiency loss due to not awarding a contract. Reserve prices and entry fees are usually used to pre-qualify bidders, like it was done in most European UMTS spectrum licence auctions. The use of lotteries where the agency sets a price and awards the contract randomly was quite common in the $1980'ies$. Especially in the US, where the allocation of spectrum licences was done by lotteries until 1994. The highly inefficient allocation of those lotteries led to the decision to design spectrum auctions, which caused a boom in research and consulting. Rationing is also a very common method in an environment where demand excess supply, for instance in equity IPO's and Central Bank Tenders[9], where bidders get only a proportion of their demand. Multi-sourcing (share auctions or split award contracts) is used when a contract will be split up in n parts and n bidders will be the winner of a $\frac{1}{n}$th share of the contract. As an example, many automobile manufacturers use more than one supplier for their components.[10] Closely related to share auctions is the analysis of multi unit auctions, where the seller sells more than one unit. Another difference to share auctions is that the bidders can have single or multi unit demand. Because all the alternative methods allocate the contract at higher prices, the probability of bankruptcy is reduced and the decision which of those mechanisms to use is faced by a tradeoff between low prices (if bankruptcy costs are low) and a low bankruptcy probability (if bankruptcy costs are high). The question is, which mechanism handles the trade-off in the best way and we shed some light on that. In addition to the methods above we investigate mechanisms that explicitly address the problem of abnormally low bids and bankruptcy, by simply by not awarding the contract to bidders with low bids. The intuition behind all those methods is, that an abnormally low bid (ALB) is

[9] Gresik (2001), more Literature in Gilbert/Klemperer.

[10] Literature in Perry and Sakovics; or defence contracts by the U.S. government and Chips by IBM, Anton and Yao (1989). An argument for multi sourcing is usually to reduce the bargaining power of the supplier.

more likely to result in a cost overrun and bankruptcy. And therefore should be checked in more detail or be should be excluded from the bidding. There is no clear definition what an ALB is, but bids are usually called ALB if the bid is a certain percentage below the average[11] or sometimes abnormally low below the second lowest bid.[12] But there is no method known to us that eliminates the lowest bid per se.[13] Usually there are not excluded but the bidders have to proof that the bid is not unrealistic. Some arithmetic procurement methods that try to handle ALB by not excluding bids being very low but by awarding bids being average were used in different countries. The goal of those methods is always the minimization of the risk of a cost overrun and of bankruptcy. Because they award the contract to some kind of average the bidders have no incentives to bid aggressive, at the cost of higher prices for the agency. For instance, in Peru[14] the average of all bids will be calculated and all bids that lie 10% above and below this average will be eliminated. The average of the remaining bids will be calculated again and the contract will be awarded to the bidder whose bid is immediately below the second average. Should none of the bids lie below the second average, the contract will go to the bid which more closely approximates the average. Similar to that rule is a method in which the winner might be the bidder with the bid closest to the average (Taiwan), or the bidder whose bid is closest to, but less than average (Italy)[15]. Also used in Italy [16] was the following method where the agency first calculated the average, then all bids above the average were excluded and the average of the remaining bids was calculated and the contract was awarded to the bidder closest to the second average. There are also some multi unit auctions with similar features. The Spanish Treasury uses a hybrid between a uniform and a discriminatory auction, where the winning bids above the average pay the average,

[11] usually 10-15%, mandatory in Belgium, France (proposed), Italy, Portugal, Romania, Spain and Greece; (EC,ALT and FIEC). The Public Works Directive 93/37/EEC of the European Union allows to investigate offers which seem to be abnormally low in relation to the work, although not allowing for the automatic rejection of the bid.

[12] In the Netherlands used to be an pre-procurement of all participating bidders with a bidding process and the disclosure of the bids, allowing bidders to withdraw their bid if it was obvious a wrong and too optimistic calculation. Although being an interesting method to cope with low bids this method raises additional concerns about competition law violations and collusion that go beyond this version of our paper. (Lupp (1993))

[13] Imagine a 100 meter race where the second fastest would win. Rumor had it that in some regions of Switzerland the lowest bid was excluded but we have no legal proof for that.

[14] Henriod and, Lantran (2000)

[15] Ioannou and Leu (1993)

[16] Lupp (1993)

otherwise they pay their respective bid.[17] Although these methods are a straightforward way to reduce ALB, we will show their drawbacks in Section 3.

Related Literature

There are just a few papers which discuss analytically the relation between bankruptcy and auctions. Zheng (2001) shows in the context of a common value selling auction, that if bidders are budget constrained and the value of the auctioned object is uncertain, it might be the case that the most budget constrained bidder is the person most likely to win the auction.[18] The reason is that if a bidder declares bankruptcy, she will loose her entire budget. As these cost of bankruptcy are smaller the smaller the budget, the bidder with the lowest budget might well be the person with the highest interest in winning the object.

The paper most close to ours is Parlane (2003). In her article, individual cost uncertainty is modelled as a general distribution with a continuous distribution density on a bounded support. In this framework she shows that the expected procurement price is larger in a FPSB-auction than in any other efficient mechanism where only the winner pays. The intuition for this result is straight forward: As the possibility of bankruptcy leads to a convex utility function, bidders behave as if they were risk loving. Only in a FPSB-auction the winning price (conditional on winning) is certain. Any other mechanism leads to uncertainty in the winning price which makes bidders bid more aggressively. Parlane concludes that this implies that a first price sealed bid auction gives the lowest probability of bankruptcy. As we show, this result is not correct in general. Furthermore, we extend the analysis by Parlane by discussing other common procurement mechanism like rationing and multisourcing.

Board (2002) uses a similar framework as Parlane (2003), although he considers a selling auction rather than a procurement auction. His mechanism design approach is similar to Zheng's (2001). He argues that limited liability makes bidding more aggressive by cutting off the downside loss. The author uses a mechanism design approach to show that the expected procurement price is lower under limited liability than under unlimited liability. He also shows that the FPSB-auction leads to the highest expected procurement

[17] Abbink et al. (2002)

[18] Che and Gale (1998) were the first to address outside financing.

price of all standard winner-pays auctions. He outlines conditions under which the FPSB-auction will lead to the lowest probability of bankruptcy. A disadvantage of his analysis is that his results depend on the assumption of decreasing absolute risk lovingness (the equivalent to increasing absolute risk aversion), which is not very intuitive. Furthermore he analyzes the case of common values.

In an experimental paper Roelofs (2002) investigates a common value framework with default. In this framework default gives the winner an opportunity to avoid the winners curse. The experiment shows that default leads indeed to more aggressive bidding, but there is no experimental support for allowing default raising more revenue as proposed by Board (2002).

In a different context, Bulow and Klemperer (2002) show that rationing and/or splitting awards can be revenue enhancing. Their result was derived in a framework with common values and asymmetric bidders. Our framework with bankruptcy gives an alternative explanation why these other forms of procurement mechanisms might be preferred to a standard auction.

Section 2 describes our model and derives the bidding strategies in a SPSB- and a FPSB-auction. Section 3 deals with alternative procurement methods and Section 4 concludes.

2 Model

A procurement agency has one tender contract to offer and all participants are risk neutral. There are n potential bidders (indexed by i) with costs being either c_i or $c_i + \Delta$, with probability ρ and $(1-\rho)$ respectively, with $\rho > 0$. The cost c is distributed on the support $[\underline{c}, \overline{c}]$, identical to all bidders. We denote $F(c)$ as the distribution and $f(c) = F'(c)$ as the density. Sometimes we use the uniform distribution to compare the results of different methods, in that case $F(c) = c$. The realization of the costs is bounded on the support $[\underline{c}, \overline{c}+\Delta]$. The term Δ can be interpreted as an error term in the calculation, common risk of the project or as financial need of the bidder, either caused by preceding projects or by other projects still in process. So the error term can reflect on one side the uncertainty at the time of the auction (private or common uncertainty) and on the other side limited liability. It is assumed that Δ is high enough ($\Delta > 0$, $\Delta < (\overline{c} - \underline{c})$) and the costs are non-negative ($\underline{c} - \Delta > 0$). If a bidder obtains a contract at a price which lies below the realized cost, the contractor declares bankruptcy. To avoid multiplicity of equilibria later

on, we assume that if a contractor goes bankrupt, she has to bear small bankruptcy cost of ϵ.[19] The order of events is as follows: (1) The seller announces the auction rules and defines the specifications of the good. (2) The bidders draw their cost out of $F(\cdot)$ and they see the level of Δ. (3) They bid in the auction relying on c_i and Δ, and the bidder with the lowest bid is declared the winner. (4) The winner observes her realized cost and either makes a profit if the payment is higher than the costs or she decides to declare bankruptcy if costs are higher than the payment. Loosing bidders all have a payoff of zero. We use costs of bankruptcy B that the agency has to bear (like delays and other accountable costs). We will call an *allocation* efficient if the bidder with the lowest cost wins. And we will sometimes refer to *efficiency* meaning the minimization of all the agency's costs (P and B). It is important that the seller has to stick to the rules set in (1) and there is no renegotiation and no resale.

If a bidder with cost c wins the contract at price p (or the payment from the bidders point of view), her expected payoff is given by

$$\pi = \begin{cases} (p-c) - (1-\rho)\Delta & \text{if} \quad p \geq c + \Delta \\ \rho(p-c) - (1-\rho)\epsilon & \text{if} \quad c + \Delta > p \end{cases} \tag{1}$$

That is if the payment is larger than costs, the bidder will fulfil the contract. If however costs are larger than the agreed upon payment, the bidder will declare bankruptcy.

The expected utility for the procurement agency is given by

$$u(p, \phi) = (1-\phi)(v - E[p]) - \phi B \tag{2}$$

where p is the sourcing price , v is the valuation of a successful implemented project[20] and ϕ is the probability that the contractor does go bankrupt. As we will show next, that probability depends on the procurement mechanism used.

Second Price Sealed Bid Auction

In a SBSB-auction the contract is awarded to the bidder with the lowest bid and the

[19] Although other authors use budgets to make the analysis more realistic, we won't use them because they do not change the effect on the bidding behavior.

[20] To avoid unrealistic equilibria we later assume that the agency will never pay more than $\overline{c} + \Delta$.

price paid is the second lowest bid.

Proposition 1 *In a SPSB-auction, in the limes of $\epsilon \to 0$, it is a weakly dominant strategy for each bidder to bid her cost.*

$$\beta_{SPSB}(c) = c \tag{3}$$

The proof is straight forward and follows the textbook analysis. The small costs of bankruptcy are assumed such that no bidder has an incentive to bid less than c. Assume that bidder i bids $b_i = c_i$ and the lowest competing bid is $b_{(2)} = min_{i \neq j} b_j$. Bidder i will win if $b_i < b_{(2)}$ and will not win if $b_i > b_{(2)}$, which gives her zero payoff.[21] The expected payoff if she wins is $\rho[b_{(2)} - c_i] + (1-\rho)[(b_{(2)} - c_i - \Delta \mid b_{(2)} > c_i + \Delta) - (\epsilon \mid b_{(2)} < c_i + \Delta)]$, which is larger than zero for ϵ small enough. Suppose now that she deviates and bids $z_i > b_i$. If $b_{(2)} < b_i < z_i$ she still gets zero payoff, if $b_i < z_i < b_{(2)}$ she still gets the same payoff as bidding b_i and if $b_i < b_{(2)} < z_i$ she looses whereas bid b_i would have won yielding a positive expected payoff. Now consider $z_i < b_i$. If $b_{(2)} < z_i < b_i$ she still gets zero payoff, if $z_i < b_i < b_{(2)}$ she still gets the same payoff as bidding b_i and if $z_i < b_{(2)} < b_i$ she wins and goes always bankrupt yielding a payoff of $-\epsilon$. So deviating from bidding $b(c_i) = c_i$ will never increase her payoff but sometimes decrease it. □

Note that the proof above must be slightly modified if the agency never pays more than $\overline{c}$ or the reservation price is $r \leq \overline{c}$. In that case the bidder with cost of $\overline{c}$ receives her bid as the payment and this would give her always a payoff of $-\epsilon$ if she wins (which is with probability zero). So bidding anything above $\overline{c}$ is an optimal bid for this bidder. But because she never wins, her expected payoff is zero and so we let her bid $\overline{c} + \epsilon$ which gives her a payoff of zero conditional on winning and having high costs.

In the proof above we assumed that bidder i knows the lowest competing bid. But the proof doesn't change if the lowest competing bid is random with some density function $f(\cdot)$. The proof is a standard Bayesian argument e.g. see Matthews (1995), working with expected profits. Thus in a second price auction, the bidder with the lowest expected costs wins the contract, i.e. the sourcing is efficient. Because we are also interested in the profit of the procurement agency, we still need the probability of bankruptcy and the expected price. In the second price sealed bid auction the winner receives the second lowest order statistic as the payment. The expected payment - if the winning bidder goes

[21] Because $f(c)$ is continuous we will neglect ties.

not bankrupt - is the expectation of the second lowest order statistic, which is also the procurement agency's expected price. This is given in the following equation, with $f_2(\cdot)$ as the density of the second lowest order statistic.

$$\begin{aligned} E[p_{SPSB}] = E[c^{(2)}] &= \int_{\underline{c}}^{\overline{c}} c f_2(c) dc \\ &= \int_{\underline{c}}^{\overline{c}} cn(n-1)F(c)(1-F(c))^{(n-2)} f(c) dc \end{aligned} \tag{4}$$

The calculation of the probability of bankruptcy is more complex, but is given in equation 2.

In a second price sealed bid auction the probability of bankruptcy is given by

$$\phi_{SPSB} = (1-\rho)[1-[1-F(\underline{c}+\Delta)]^n] \tag{5}$$

Proof: From the point of view of the contracting agency, the probability of not being served is equal to $(1-\rho)$ times the probability that the second lowest cost is less than Δ away from the lowest cost. Formally, the latter term is the probability that $c^{(2,n)} - c^{(1,n)} < \Delta$, where $c^{(i,n)}$ is the i'th (lowest) order statistic from n draws. Let $f_1(c)$ be the density of the lowest order statistic and let $f_{1,2}(c)$ be the density of the second lowest order statistic conditional on c is the lowest order statistic.

$$\begin{aligned} Prob[c^{(2,n)} - c^{(1,n)} < \Delta] &= \int_{\underline{c}}^{\overline{c}} f_1(c) \int_{c}^{c+\Delta} f_{1,2}(z) dz dc \\ &= \int_{\underline{c}}^{\overline{c}} nf(c)[1-F(c)]^{(n-1)} \int_{c}^{c+\Delta} \frac{n(n-1)f(c)f(z)[1-F(z)]^{(n-2)}}{nf(c)[1-F(c)]^{(n-1)}} dz dc \\ &= (1-[1-F(\underline{c}+\Delta)]^n) \end{aligned} \tag{6}$$

So the expected utility for the agency is

$$E[u(p,\phi)] = (1-\phi_{SPSB})(v - E[p_{SPSB}]) - \phi_{SPSB} B \tag{7}$$

Remark 1 Note, that the bidding strategy, the expected price and the probabilities of bankruptcy would be same in an English auction. □

First Price Sealed Bid Auction

In a FPSB-auction the bidder with the lowest bid wins and the payment she receives is the winning bid. Here multiplicity of equilibria is not a problem. Therefore we set $\epsilon = 0$. Furthermore, we distinguish between two cases, whether n is small or large.

Proposition 2 *In a FPSB-auction, for $n \geq 1 + \frac{1}{\Delta \rho f(c)}$ (n large), an equilibrium exists where a bidder with cost c will bid the expectation of the lowest cost of the (n-1) remaining bidders, conditional on her cost being the lowest, which is $E[c^{(1),(n-1)}|c^{(1),(n-1)} > c]$ or*

$$\beta_{FPSB}(c) = \frac{1}{1 - G(c)} \int_c^{\overline{c}} zg(z)dz = c + \int_c^{\overline{c}} \frac{(1 - G(z))}{(1 - G(c))} dz \tag{8}$$

with the boundary condition $b(\overline{c}) = \overline{c}$ and with $(1 - G(c)) = (1 - F(c))^{(n-1)}$.

The bidding function for the uniform distribution is given is Figure 3.

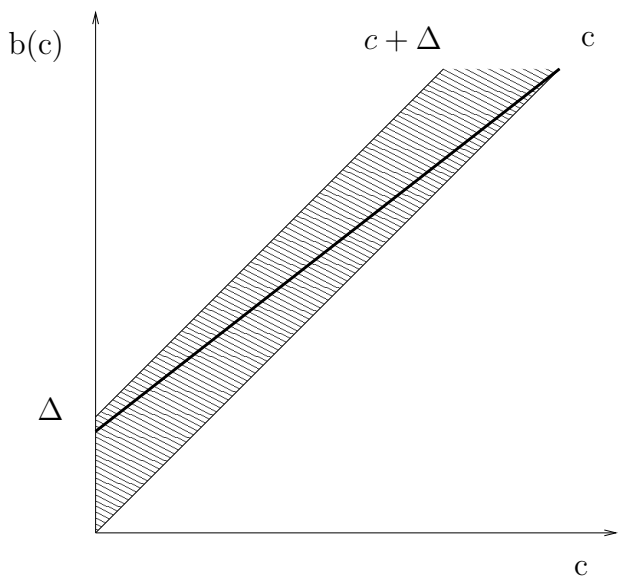

Figure 3: FPSB-auction (n large), bidding function

Proof: Suppose all bidders ($j \neq i$) follow the bidding strategy β_{FPSB} given in Proposition 2. We will argue that in that case it is optimal for bidder i to follow β_{FPSB} also. Note that if a bidder bids according to β_{FPSB}, her bid lies below $c+\Delta$ but above c. First we show that under the assumption that a bidder with cost c bids less than $c+\Delta$, it is indeed optimal for her to bid according to the equilibrium strategy. At equilibrium a bidder with cost c chooses a cost $\hat{c}$ and its corresponding bid $\beta(\hat{c})$. Formally such a bidder maximizes the following expression with respect to $\hat{c}$:

$$\pi(\hat{c}, c) = \rho(\beta(\hat{c}) - c)(1 - F(\hat{c}))^{(n-1)} \tag{9}$$

where the right term is the probability that all other bidders have costs larger than $\hat{c}$. For an easier derivation of the bidding strategy we denote $(1 - G(c)) = (1 - F(c))^{(n-1)}$. The derivative of (9) with respect to $\hat{c}$ gives the first order condition, which is the following differential equation

$$\beta'(\hat{c})(1 - G(c)) + (\beta(\hat{c}) - c)(-g(\hat{c})) = 0 \tag{10}$$

where $-g(c) = d(1 - G(c))/dc$

At a symmetric equilibrium $c = \hat{c}$ so equation (10) can be rewritten as

$$\frac{d}{dc}(1 - G(c)\beta(c)) = -cg(c) \tag{11}$$

integrating both sides yields

$$\beta_{FPSB}(c) = \frac{1}{1 - G(c)} \int_c^{\overline{c}} zg(z)dz = E[c^{(1),(n-1)} | c^{(1),(n-1)} > c] \tag{12}$$

with the integration constant $C = 0$ for $\beta(\overline{c}) = \overline{c}$, and that is the desired solution which can also be written as a result of integration by parts

$$\beta_{FPSB} = c + \int_c^{\overline{c}} \frac{(1 - G(z))}{(1 - G(c))} dz = c + \int_c^{\overline{c}} \frac{(1 - F(z))^{(n-1)}}{(1 - F(c))^{(n-1)}} dz \tag{13}$$

which is unique because β is a Nash-Equilibrium and increasing in c. □

For β_{FPSB} to be smaller than $c + \Delta$, we need $\int_c^{\overline{c}} \frac{(1-F(z))^{(n-1)}}{(1-F(c))^{(n-1)}} dz < \Delta$ as a minimum requirement, which is true for $\frac{1}{nf(c)} < \Delta$.

In a second step we show that if everyone else behaves according to this strategy, it is indeed not optimal to bid more than $c + \Delta$. Assuming that a bidder bids such that she never goes bankrupt yields the profit function:

$$\pi(\hat{c}, c) = (\beta(\hat{c}) - c + \Delta(\rho - 1))(1 - F(c))^{(n-1)} \tag{14}$$

Maximizing this expression with respect to $\hat{c}$ yields:

$$\beta(c) = \frac{\beta'(c)(1 - F(c))}{(n-1)f(c)} + c - \Delta\rho + \Delta \tag{15}$$

showing that the solution for the optimal bid $\beta(c)$ is smaller than $c + \Delta$ whenever

$$\rho \geq \frac{\beta'(c)(1 - F(c))}{(n-1)f(c)\Delta} \tag{16}$$

Using that $(1 - F(c)) \leq 1$ and $\beta'(c) \leq 1$ we can rewrite the condition as

$$\rho \geq \frac{1}{(n-1)f(c)\Delta} \tag{17}$$

or reformulated

$$n \geq \ 1 + \frac{1}{\Delta\rho f(c)} \tag{18}$$

which is the condition in proposition 2. This implies that the probability of having high costs must be sufficiently small, as otherwise bidding below the high cost level would not be profit maximizing.

In the FPSB-auction (n large) the expected payment for a bidder i with cost $c_i = c$ is her bid times the probability of winning.

$$E[p(c_i = c)] = (1 - G(c))\beta(c) = \int_c^{\overline{c}} zg(z)dz \tag{19}$$

so the ex ante expected payment for a bidder is

$$E[p(c)] = \int_{\underline{c}}^{\overline{c}} [\int_{c}^{\overline{c}} zg(z)dz]f(c)dc \tag{20}$$

Interchanging the order of integration we obtain that

$$E[p(c)] = \int_{\underline{c}}^{\overline{c}} z[\int_{\underline{c}}^{z} f(c)dc]g(z)dz = \int_{\underline{c}}^{\overline{c}} z[F(z)]g(z)dz \tag{21}$$

And the expected price for the procurement agency is n times the ex ante expected payment for an individual bidder, so

$$\begin{aligned} E[p_{FPSB}] &= n \int_{\underline{c}}^{\overline{c}} z[F(z)]g(z)dz \\ &= \int_{\underline{c}}^{\overline{c}} nc(n-1)F(c)(1-F(c))^{(n-2)} f(c)dc \end{aligned} \tag{22}$$

with $G'(c) = g(c) = f(c)(n-1)(1-F(c))^{(n-2)}$. So the expected price in the FPSB-auction is the same in the SPSB-auction, if the winning bidder goes not bankrupt.

As β_{FPSB} is increasing and continuous, in equilibrium the bidder with the lowest cost submits the lowest bid and wins the auction. Thus also a first price auction is efficient. However, note that under the condition of n large, each bidder will bid less than $c + \Delta$. Therefore, from the point of view of the contracting agency, the probability of not being served is given in equation 23.

In a first price sealed bid auction for n large the probability of bankruptcy is

$$\phi_{FPSB} = 1 - \rho \tag{23}$$

ϕ_{FPSB} is larger than the probability under a SPSB-auction given in equation (5). It also does not depend on the cost distribution. The expected utility of the procurement agency in the FPSB-auction is then

$$E[u(p, \phi)] = \rho(v - E[p_{FPSB}]) - (1 - \rho)B \tag{24}$$

Remark 2 The bidding strategy in a Dutch auction for n large would also to bid the expectation of the cost of the lowest competitor. □

The interpretation of the n large case is the following. The competition in the auction (a high number of bidders or a high ρ) must be sufficiently high to force all bidders to bid very aggressive (below the threshold $c + \Delta$). That means every bidder adds less than Δ to his costs as a profit margin, conditional on winning as in equation (8). Raising the bid above the threshold would lower the expected utility of the bidder because the gain in payoff is smaller than the loss on the probability of winning. But if the competition is not hard enough (n small) then that consideration no longer holds and bidders with low costs can increase their expected payoff by bidding above the threshold. Which increases their profit margin at the loss of a lower probability of winning. So next we consider is the case of n small:

Proposition 3 *In a first price sealed bid auction, if n is small, there exists a $\hat{c} \in [\underline{c}, \overline{c}]$ and an equilibrium bid function $\beta_{FPSB}(c)$ with the following properties: If $c \leq \hat{c}$, than $\beta_{FPSB}(c) \geq c + \Delta$ and $\beta'_{FPSB}(c) > 0$. If $c > \hat{c}$, then $c \leq \beta_{FPSB}(c) < c + \Delta$ and $\beta'_{FPSB}(c) > 0$. Thus the bid function has a jump at $c = \hat{c}$.*

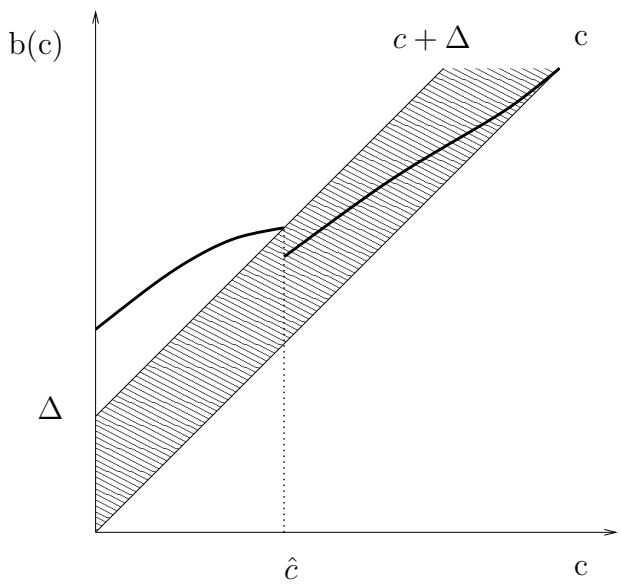

Figure 4: FPSB-auction (n small), bidding function

As the first step, it is shown that for n small no equilibrium bid function exists where

either all bids $b(c)$ are larger or all bids are smaller than $c + \Delta$. Note that is a dominant strategy for a bidder with costs $\overline{c}$ to bid $b(\overline{c})$.[22] So there can't exist an equilibrium where all bidders bid above $c+\Delta$. Because an equilibrium where all bids lie below $c+\Delta$ is given by n large, the bids for n small have to lie above and below the threshold. The rest of the proof proceeds in three steps. We use revealed preferences to proof that the bid function below and above $\hat{c}$ is weakly increasing and conclude that the bid function has a jump. First, we show that if $c_1 < c_2 < \overline{c}$ and the equilibrium bids $b(c_i), i = 1, 2$ are greater than $c_i + \Delta$ then it must be the case that $b(c_1) < b(c_2)$. The utility of a bidder with cost c_i and a bid $b(c_i)$ above $c_i + \Delta$ is $(b(c_i) - c_i - \Delta(1-\rho))Q(b(c_i))$. With $Q(b(c_i)$ as the probability of winning with bid $b(c_i)$. So for $c_1 + \Delta < c_2 + \Delta < b(c_i)$ the following inequalities hold where both bidders prefer to bid according to their bidding strategy instead of bidding the others strategy:

$$\begin{aligned} &(b(c_1) - c_1 - (1-\rho)\Delta)Q(b(c_1)) > (b(c_2) - c_1 - (1-\rho)\Delta)Q(b(c_2)) \\ &(b(c_2) - c_2 - (1-\rho)\Delta)Q(b(c_2)) > (b(c_1) - c_2 - (1-\rho)\Delta)Q(b(c_1)) \end{aligned} \tag{25}$$

Solving these two inequalities leads to

$$(c_1 - c_2)(Q(b(c_2)) - Q(b(c_1))) > 0 \tag{26}$$

and because $c_1 < c_2$, it follows that $Q(b(c_1)) > Q(b(c_2))$ and this implies $b(c_1) < b(c_2)$, so the bid function is increasing.

Second, it is shown that if $c_1 < c_2$ and $b(c_i) < c_i + \Delta, i = 1, 2$, then $b(c_1) < b(c_2)$. The utility of a bidder with cost c_i and a bid $b(c_i)$ below $c_i + \Delta$ is $\rho(b(c_i) - c_i)Q(b(c_i))$. So for $b(c_i) < c_1 + \Delta < c_2 + \Delta$ the following inequalities hold where both bidders prefer to bid according to their bidding strategy instead of bidding the others strategy:

$$\begin{aligned} &\rho(b(c_1) - c_1)Q(b(c_1)) > (\rho(b(c_2) - c_1)Q(b(c_2)) \\ &\rho(b(c_2) - c_2)Q(b(c_2)) > \rho(b(c_1) - c_2)Q(b(c_1)) \end{aligned} \tag{27}$$

[22] Assume that this bidder will bid as in a SPSB-auction with $\epsilon \to 0$

Solving these two inequalities leads to

$$\rho(c_2 - c_1)(Q(b(c_1)) - Q(b(c_2))) > 0 \tag{28}$$

and because $\rho > 0$ and $c_1 < c_2$, it follows that $Q(b(c_1)) > Q(b(c_2))$ and this implies $b(c_1) < b(c_2)$, so the bid function is increasing.

In a third step it is shown that if there exist a couple (c_1, c_2) with $b(c_1) > c_1 + \Delta$ and $b(c_2) < c_2 + \Delta$. We will show this for the case $b(c_2) < c_1 + \Delta < c_2 + \Delta < b(c_1)$, (although there are more rankings that yield the same results.)

$$\begin{aligned} &(b(c_1) - c_1 - (1-\rho)\Delta)Q(b(c_1)) > (\rho(b(c_2) - c_1)Q(b(c_2)) \\ &\rho(b(c_2) - c_2)Q(b(c_2)) > (b(c_1) - c_2 - (1-\rho)\Delta)Q(b(c_1)) \end{aligned} \tag{29}$$

Solving these two inequalities leads to

$$(c_1 - c_2)(Q(b(c_1)) - \rho Q(b(c_2))) < 0 \tag{30}$$

and because $\rho > 0$ and $c_1 < c_2$, it follows that the *expected* probability of winning with $b(c_1)$ must be sufficiently high that $Q(b(c_1)) > \rho Q(b(c_2))$.[23] So the bid function has a jump at $\hat{c}$ and we will call a bidders with costs smaller than $\hat{c}$ bidders with low costs and all above bidders with high cost.

The intuition behind this result is the following: While high types still have an incentive to bid below $c + \Delta$ to win more often, low cost bidders can abstain from the aggressive bidding in the presence of only a few bidders. While shading their bid below $c + \Delta$ would raise the probability of winning for a low cost bidder slightly, it would reduce the profit if she wins substantially. Therefore low costs bidders won't do this and will bid above $c + \Delta$. The results of proposition 3 are different to that of the existing literature. A bidding function with a jump like this is not found in other papers, maybe because the result is caused by the discrete error term in our binary model. If the error term follows a continuous distribution between 0 and Δ then the bidders will follow a monotone, symmetric equilibrium bidding function, as shown in Parlane (2003).[24] But the jump nicely

[23] The probability of winning with $b(c_1)$ is sufficiently high if there are only a few bidders present or ρ is small enough.

[24] Under some assumptions.

shows the two effects bankruptcy causes. One, bankruptcy gives incentives for bidders with disadvantages to bid more aggressive. Two, to avoid default risk the agency has to pay a premium.

Because we can't derive the bidding function we also can't say anything about the expected probability of bankruptcy. But we can calculate the maximal probability of bankruptcy. First, remember that a bidder with cost $\overline{c}$ will bid $\overline{c}$. Now, if the bid of the bidder with $c = \hat{c}$ is above $\overline{c}$, then she has zero probability of winning. So reducing the bid to $\overline{c}$ will lead to a tie with the $\overline{c}$ cost bidder. She then clearly has an incentive to slightly underbid $\overline{c}$ by a small amount ϵ ($\epsilon > 0$) to increase her probability of winning (first order effect). Giving away a profit that is only of second order. So her bid will be between $\hat{c} + \Delta \leq b(\hat{c}) < \overline{c}$. To calculate the highest possible probability of bankruptcy for n small, the bidder with $\hat{c}$ has to bid as low a possible, which is $\hat{c} + \Delta$. This is ϵ below $\overline{c}$ and the bidders with costs higher than $\hat{c}$ will go bankrupt with probability $(1 - \rho)$. She bids $b(\hat{c}) = \hat{c} + \Delta = \overline{c} - \epsilon < \overline{c}$, which gives $\hat{c} = \overline{c} - \Delta - \epsilon$, or $\hat{c} = \overline{c} - \Delta$ for $\epsilon \to 0$. So the minimum probability of bankruptcy is given by the bidder with cost $\hat{c}$, which is the probability that no competing bidder has costs below $\overline{c} - \Delta$

$$(1 - \rho) Prob[c^{(1)} > \overline{c} - \Delta] = (1 - \rho)[1 - F(\overline{c} - \Delta)]^n \tag{31}$$

which goes to zero for $\Delta \to 0$, because if the uncertainty vanishes, no bankruptcy can occur. So also for the n small case there is always a positive probability of bankruptcy, although smaller than in the other standard auctions.

Comparison between the SPSB- and FPSB-auction

(i) In both auction formats (for n large) the winner is always the most efficient bidder. Because both formats are standard auctions by definition this result is not surprising. This result does not hold for a small number of bidders, where the allocation can be inefficient (see Figure 4). So if the agency is interested in allocation efficiency there is a trade-off between allocating the good to the most efficient bidder and price and on the other side the probability of bankruptcy. If the agency prefers to allocate the good to the most efficient bidder and has low B, she should induce enough entry into an auction to make n large enough.

(ii) The expected price in both auction formats (for n large) is the same. In the SPSB-auction the expected price is the expectation of the second lowest order statistic. The price itself is a random variable and is distributed like the second lowest order statistic. In the FPSB-auction the expected price is the expected value of the second lowest bid under the assumption that the bidders cost c is the lowest cost.

$$E[p_{SPSB}] = E[p_{FPSB}] \tag{32}$$

So also in the FPSB-auction the expected price is distributed like the second lowest order statistic, but it is certain for the winner. The equality of the expected prices might be caused by the discrete error term in our framework, and therefore be considered as an extreme case of bankruptcy. If we make the error term continuous, the prices in the FPSB-auction are at least the same (as we have shown) or larger as in the SPSB-auction.[25] The intuition behind this result is straight forward. The option to declare bankruptcy after the auction changes the preferences of the bidders from risk neutrality into some kind of risk lovingness (see Figure 2). As the first price sealed bid auction incorporates less risk conditional on winning (because of the certain price), the competition is softer. While the bidding strategy in a second price sealed bid auction is unaffected by the bidders attitude to risk. Because in our framework this effect does not affect the expected price, any difference in the auction formats can be explained by the difference in the price distribution. The expected price for a small number of bidders will lie above that of the FBSP-auction (for n large) and SPSB-auction for obvious reasons. So if the agency prefers a lower price she should again induce entry.

(iii) Assuming that the expected prices in both formats are equal, the probabilities of bankruptcy differ conditional on winning with costs c:

$$\phi_{SPSB} < \phi_{FPSB} \tag{33}$$

In her article Parlane (2002) derives the result that the expected price in the FPSB-auction is higher than in the SPSB-auction, for a continuous error term distribution. And upon this result she speculates that the probability of bankruptcy is smaller in the

[25] see more details in Board (2002).

FPSB-auction. However this is not true in general, because for our case with a discrete error term distribution and for n large, the probabilities are reverse. Board (2002) shows, using the intuition with risk loving behavior, that if the prices are the same and the general cost distribution function is convex, then the probability of bankruptcy is higher in the second price sealed bid auction.[26] However in our framework the realization of the cost distribution $H(\cdot)$ in the relevant region is *quasi*-concave, so the probability of bankruptcy is lower in the second price sealed bid auction. This can also be shown by Jensens Inequality: Conditional on winning with cost c the probability of bankruptcy in the FPSB-auction is $H(p_{FPSB})$ while in a SPSB-auction $\int H(p_{SPSB})f(p_{SPSB})dp_{SPSB}$. Due to Jensens Inequality if $H(\cdot)$ is concave the probability of bankruptcy is smaller in the second price sealed bid auction. To see that the probability of bankruptcy in a FPSB-auction is smaller for a small number of bidders just compare equation (23) to equation (31). If the agency has to bear high costs of bankruptcy B she might want to deter entry into a FPSB-auction. To see if the probability is also smaller than in a SPSB-auction, compare it to equation (5). First, assume that the probability is smaller, then the following inequality should hold:

$$[1 - F(\overline{c} - \Delta)]^n + [1 - F(\underline{c} + \Delta)]^n < 1 \tag{34}$$

which is always true for $n \geq 2$ because the left hand side is a convex combination and decreases in n.

(iv) As the expected price $E[p]$ is the same in both auction formats and the probability of bankruptcy differs, payoff equivalence for the bidders breaks down. Risk loving (and also risk avers) behavior usually leads to different expected prices in different auction formats, but we have a special case with identical expected prices here (which might be explained by the discrete error term in our framework). Note however that the payoff equivalence theorem does not break down because of the shift to risk loving behavior, but due to the differences in the price distributions. This leads to the direct conclusion that the revenue equivalence (here: utility equivalence) also breaks down because of the differences in the price distributions. So the expected utility for the procurement agency is higher in the

[26] One drawback of the analysis in Board (2002) is that he assumes decreasing absolute risk lovingness, and this assumption is (like IARA) not very intuitive. Our simpler framework allows us the direct calculation of the results and so we can derive more general results regarding risk preferencs.

SPSB-auction.

$$u_{SPSB} > u_{FPSB} \tag{35}$$

If the number of bidders is small the results are ambiguous, depending on the costs of bankruptcy B. As mentioned before there is a tradeoff between the price and allocation efficiency and the costs of bankruptcy.

Remark 3 If the procurement agency is risk avers and maximizes revenue, this model can lead to the opposite conclusion as the standard auction literature, that the procurement agency might prefer the SPSB-auction. □

Remark 4 The bidders also prefer the SPSB-auction over the FPSB-auction for n large. □

Remark 5 The bidding strategies in both formats converge for $n \to \infty$. □

Extensions

(i) *Reserve Price*

Introducing a reserve price r below the agency's maximal willingness to pay can increase the agency's utility in a standard auction. Excluding bidders with costs higher than the reserve price leads on one side to a positive probability that no sale occurs, but on the other side to lower expected prices. If the agency sets the reserve price optimally the exclusion of some bidders and the resulting loss can be smaller than the gain of lower expected prices. An other main result of an introduction of a reserve price is also that in a standard private value model the expected price in a SPSB- and a FPSB-auction is the same. But we will show that this result no longer holds in our framework, because bankrupt bidders add one additional effect of the exclusion. The agency not only lowers the price, but can also affect the probability of bankruptcy, because only high cost bidders with a higher probability of bankruptcy are excluded, so a reserve price works as a coordinating device. Note, that a reserve price usually reduces efficiency due to not awarding the contract. But here this must not be true, because of a reduction in the probability of bankruptcy. We also make use of the fact that the agency suffers costs of bankruptcy B (for which she is accountable like delays or costs for a new contractor or efficiency losses). Because the optimal setting of the reserve price depends on the bankruptcy cost

of awarding the contract to a bidder that goes bankrupt. The probability that the agency suffers bankruptcy cost is the probability of a sale times the probability of bankruptcy.

The bidding strategy in the SPSB-auction with bankruptcy and a reserve price is straightforward. Because the weakly dominant strategy of bidding the cost c is independent of the number of bidders, the exclusion of bidders has no effect on the bidding strategy. Therefore all i bidders with cost below the reserve price will bid c_i. Bidders with c larger than the reserve price will never bid, because they would always make a loss. A bidder with cost $c = r$ only wins if no other bidder bids, even if she bids less than r, so her bid will be as high as possible.[27] The expected ex-ante price for a bidder with cost $c \leq r$ that stays solvent is

$$E[p(c)] = r(1 - G(r)) + \int_c^r zg(z)dz \tag{36}$$

the ex-ante expected price is

$$E[p(c)] = \int_{\underline{c}}^r r(1 - G(r))f(c)dc + \int_{\underline{c}}^r f(c) \int_c^r zg(z)dzdc \tag{37}$$

Interchanging the order of integration yields

$$E[p(c)] = r(1 - G(r))F(r) + \int_{\underline{c}}^r zg(z)F(z)dz \tag{38}$$

which is lower than without a reserve price, but the contract will not be awarded with some positive probability.[28]

[27] Note, that it is important that the agency can credible commit to not selling the good in case no one bids below the reserve. Otherwise it will face a problem similar to the durable goods monopoly problem.

[28] The expected price in FPSB-auction for n large is the same (see Appendix), which has to be the case, because in both auctions the highest bidder receives zero payoff and they yield the same allocation rule (Myersons lemma). But the probability of bankruptcy ϕ differs, so the utility for the agency will again differ (it will be lower). Also the critical number of bidders for n large is different to the auction without a reserve price.

The probability of bankruptcy in a SBSP-auction is the following:

$$\phi_{SPSB,r} = (1-\rho)Prob[c^{(1,n)} < r]\times$$

$$\{Prob[c^{(2,n)} - c^{(1,n)} < \Delta]Prob[c^{(2,n)} < r] \tag{39}$$

$$+ Prob[r - c^{(1,n)} < \Delta]Prob[c^{(2,n)} > r]\}$$

The probability of bankruptcy for the winning bidder with costs below r is larger in a SPSB-auction with a reserve price because the expected price is smaller due to the cut-of at the reserve price (the last two terms of the equation), but it can be smaller for the agency due to not awarding the contract at all (first term of the equation).[29]

The utility of the agency for all standard auctions (SPSB and FPSB for n large) that yield the same expected price and the same efficient allocation is

$$u(r,k) = \sum_{i=1}^{n} \int_{\underline{c}}^{r} (1-G(c_i))f(c_i)((1-\phi_i)(v - c_i - \frac{F(c_i)}{f(c_i)} - \frac{\phi_i}{(1-\phi_i)}B)dc_i \tag{40}$$

with $\int_{\underline{c}}^{r}(1-G(c_i))f(c_i)dc_i$ is the probability of a sale to bidder i. For the distribution F(c) we assume $(v - c_i - \frac{F(c_i)}{f(c_i)} - \frac{\phi_i}{(1-\phi_i)}B)$ decreases in c. For the optimal r^* set $(v - r - \frac{F(r)}{f(r)} - \frac{\phi_i}{(1-\phi_i)}B) = 0$ because the maximization problem is how many bidders to exclude and what bankruptcy probability to create. Note, that the optimal r does not directly depend on the number of bidders because the optimal marginal cost of r^* does not depend on the number of bidders. Only indirect over the probability of bankruptcy. The optimal reserve price is an optimization problem of equation (40), a trade off between excluding bidders (and lower prices and higher bankruptcy probabilities) and not awarding the contract (at

[29]The probability of bankruptcy in a FPSB-auction is $(1-\rho)$ times the probability that the lowest order statistic is below r, which is $1-(1-F(r))^n$. It is again larger than the probability of bankruptcy in a SPSB-auction. To compare the optimal reserve price in both auction formats we have to check in which auction the reserve is higher for the same utility or in which auction format the utility is higher for the same reserve. Intuition leads to the result that the reserve price in the SPSB-auction can be higher because at the same price (same marginal bidder) the probability of bankruptcy is smaller and therefore the utility of the agency is higher. So a SPSB-auction that yields the same utility for the agency as a FPSB-auction has a higher reserve price. This is important if entry into an auction is endogenous.

higher prices and lower bankruptcy probabilities). Which leads to

$$r = v - \frac{F(c)}{f(c)} - \frac{\phi}{1-\phi}B \tag{41}$$

In a SPSB-auction the introduction of a reserve price has no effect on the bidding behavior and is therefore not suitable as a coordinating device. It only alters the distribution of the second lowest bid which determines the price. So introducing a reserve price into an auction (even without B) can increase the agency's utility. If the agency suffers cost B from a bankrupt winner, then the optimal reserve price decreases with an increase of bankruptcy costs ($B > 0$). But note that in some formats (SPSB-auction) the probability of bankruptcy is also affected by a lower reserve price and therefore this effect works into the other direction.

Proof: A solvent bidders expected price is

$$E[p(c)] = c(1 - G(c))) + \int_c^r (1 - G(z))dz \tag{42}$$

the ex ante expected price for a each solvent bidder is

$$\int_{\underline{c}}^r E[p(c)]f(c)dc = \int_{\underline{c}}^r f(c) \int_c^r (1 - G(z))dzdc + \int_{\underline{c}}^r (1 - G(c))cf(c)dc \tag{43}$$

Interchanging the order in the first term of the right hand side yields

$$\int_{\underline{c}}^r E[p(c)]f(c)dc = \int_{\underline{c}}^r (1 - G(z)) \int_{\underline{c}}^z f(c)dcdz + \int_{\underline{c}}^r (1 - G(c))cf(c)dc \tag{44}$$

or

$$\int_{\underline{c}}^r E[p(c)]f(c)dc = \int_{\underline{c}}^r (1 - G(z))F(z)dz + \int_{\underline{c}}^r (1 - G(c))cf(c)dc \tag{45}$$

which is (after changing the integration variable)

$$\int_{\underline{c}}^r E[p(c)]f(c)dc = \int_{\underline{c}}^r (1 - G(c))f(c)(\frac{F(z)}{f(c)} + c)dc \tag{46}$$

Multiplying with $(1 - \phi)$ leads to expected prices including bankruptcy. Summing over i bidders (restoring subscripts) yields the sum of all bidders expected prices. The proba-

bility that the agency sells to bidder i is

$$\int_{\underline{c}}^{r} (1 - G(c_i)) f(c_i) dc_i \tag{47}$$

The probability of solvency times the sum of the agency's value and the same computation for the costs of bankruptcy leads to the desired equation.

(ii) *entry fees*

The effect of entry fees in a standard auction without bankruptcy is the same as a reserve price. But in our framework this equivalence also breaks down. Assume that all bidders have to pay a fixed fee k for the right to participate. So bidders with costs close to $\overline{c}$ will not place a bid, because the payoff would be negative if they win (k is not a sunk cost). We call the last bidder that wants to participate the marginal bidder c_o and she is indifferent between bidding and not participating, so her payoff is zero. This marginal bidder is the same as the bidder with cost r in the reserve price auction. The following analysis is therefore similar. Let us assume for simplicity that the agency will never pay more than $\overline{c} + \Delta$. Then the expected ex-ante price for a bidder with cost $c \leq c_o$ that stays solvent is

$$E[p(c)] = (\overline{c} + \Delta)(1 - G(c_o)) + \int_{c}^{c_o} z g(z) dz \tag{48}$$

which is slightly higher than with a reserve price because if only one bidder participates she receives not the reserve price but the highest price the agency is willing to pay, $\overline{c} + \Delta$. The same reason leads to a lower probability of bankruptcy

$$\phi_{SPSB,K} = (1 - \rho) Prob[c^{(1,n)} < c_o] Prob[c^{(2,n)} - c^{(1,n)} < \Delta] Prob[c^{(2,n)} < c_o] \tag{49}$$

Even if the reserve price and the entry fee lead to the same marginal bidder $r = c_o$, the price in the entry fee case is higher and the probability of bankruptcy $\phi_{SPSB,K}$ is slightly smaller than $\phi_{SPSB,r}$.

(iii) *asymmetries*

Several types of asymmetries can be present. One, bidders have different distributions $F(\cdot)$ of the costs. Two, bidders have different uncertainties of their cost realization ρ (for

instance an information advantage of an incumbent). And three, bidders have different Δ's (for instance a cost advantage of an incumbent). The analysis of different Δ and ρ is similar, therefore we investigate only one of those. We model the asymmetries here over different distributions, so that all three asymmetries from above are covered. First we investigate different ρ (Δ)where we assume that there are two types of $\rho \epsilon \{\rho_l, \rho_h\}$ ($\Delta \epsilon \{\Delta_l, \Delta_h\}$) with $\rho_h > \rho_l$ ($\Delta_h > \Delta_l$). For simplicity assume that ρ_l (Δ_h) equals the ρ (Δ) from the previous analysis. So for some bidders the uncertainty is the same and for others the situation is better (FOSD, shift in the distribution). In a SPSB-auction it is still a dominant strategy for every bidder to bid her cost, independent of ρ or Δ. So the allocation and the distribution of $c^{(2,n)}$ will still be the same. But the profit of the winning bidder can be higher and the probability of bankruptcy will depend on the distribution of ρ (Δ), conditional on the ρ-type (Δ-type) of the winning bidder. So if some bidders have higher ρ's (lower Δ's) the agency benefits because of lower bankruptcy probabilities.

In a FPSB-auction the analysis is different. It is very hard to characterize an equilibrium in a FPSB-auction with asymmetric bidders. For an easier analysis assume there are only two bidders, one with a $\rho_i = \rho < 1$ (Δ) and one with a $\rho_{-i} = 1$ ($\Delta = 0$). We make this assumption because with this kind of distribution an equilibrium can be derived. Here the cost distribution of the advantaged (strong or more efficient) bidder first order stochastically dominates (FOSD) the (expected) distribution of the second bidder. We also assume that the costs are uniformly distributed on $[\underline{c}, \overline{c}]$. We begin our analysis assuming different bidding functions. We will follow the argumentation of Maskin and Riley (2000a), where they derive an equilibrium that has the following property: If one players costs are distributed lower than the other in some way of FOSD, then the advantaged bidders bid's are distributed the same way. This result leads to the conclusion that the weakness of the one bidder makes the bidding more aggressive. The intuition behind this result is that when a bidder is stronger, the competitor bids more aggressive. As shown in Maskin and Riley (2000b) the strong bidder bids more for each possible cost. Combining the last two results, the strong bidder has lower equilibrium bids, but bids more for each realization of her costs. So the strong bidder can sometimes lose, even if she has a lower cost. In contrast to the SPSB-auction where the strong (more efficient) bidder always wins. The intuition above suggest that the expected price in a FPSB-auction is lower than in a SPSB-auction on average, because only the FPSB-auction gives the 'right' incentives to bid aggressive. That also leads to the conclusion, that the payoff in a SPSB-

auction is higher for a strong bidder than in a FPSB-auction. And higher for the weak bidder in a FPSB-auction compared to the SPSB-auction. In the SPSB-auction there will never be a bankruptcy, because the strong bidder ($\Delta = 0$) always wins. Whereas the FPSB-auction can not only be inefficient in terms of allocation, but also leads to a positive probability of bankruptcy. Again, the agency might not want to buy via a FPSB-auction, because if she has to suffer large bankruptcy costs B or has a high valuation v, the FPSB-auction would be suboptimal.

iv) *common values (costs)*

In many environments the cost of the bidders are common or almost common costs.Here we follow the analysis of Bulow and Klemperer (2002) and investigate the almost common cost case for symmetric and asymmetric bidders in an English (open) auction.[30] We investigate the three bidder case and compare the results of single unit and split-award actions. But the results can be generalized to more than three bidders. In a common cost environment the winner is the bidder who has the most optimistic signal. Conditional on winning this signal would be too high on average and therefore the winning bidder will lose money on average, because the realization of the cost is the average of the signals. (this *winners curse* increases in the number of bidders). To avoid that, bidders will reduce their bids via updating their beliefs about the 'true' common cost with the information of the bidders that drop out of the auction. So the knowledge of the winners curse and the updating leads to more cautious bidding. On the other side, allocations or payment rules that reduce the winners curse will reduce the incentive to bid cautiously and therefore will lead to more aggressive bidding. So split awards or average rules can lead to lower prices and higher bankruptcy probabilities, the opposite result of the private cost case.

Let c_i be the signal for bidder i and $\overline{c}$ be the highest possible signal. The expected cost C for each bidder has a common cost part and a private cost part Δ and ρ, so they are almost common costs.

$$\begin{aligned} C_1 &= c_1 + c_2 + c_3 + \Delta_1(1-\rho_1) \\ C_2 &= c_1 + c_2 + c_3 + \Delta_2(1-\rho_2) \\ C_3 &= c_1 + c_2 + c_3 + \Delta_3(1-\rho_3) \end{aligned} \tag{50}$$

[30]The analysis is a 1:1 copy of the analysis of Bulow and Klemperer, so the proofs can be found there.

with $C = \sum_{i=1}^{n} c_i$. We consider two different cases the symmetric and the asymmetric case. In the first $\Delta_1 = \Delta_2 = \Delta_3 = \Delta$ and $\rho_1 = \rho_2 = \rho_3 = \rho$. And the latter $\Delta_1 < \Delta_2 = \Delta_3 = \Delta$ or $\rho_1 > \rho_2 = \rho_3 = \rho$. Because the analysis of different ρ's or Δ's is the same, we investigate only different Δ's, the results carry over to different ρ's. There are either one or two units for sale (selling two equal splits instead of two units is the same analysis) and bidders demand is one (restricted to bid only for one unit or one share of a unit). We assume hazard rates $h_i = \frac{f(c_i)}{F(c_i)}$ are increasing, as in Bulow and Klemperer. In the symmetric case a bidder with the highest signal $c^{(3)}$ quits at the price where she is indifferent about winning if both opponents drop out at that price, which is $3c^{(3)}$. Like in the private cost auction bidders ignore Δ and bid below the threshold of not going bankrupt, in this case below $3c^{(3)} + \Delta(1-\rho)$. The next highest bidder quits when $p = c^{(3)} + 2c^{(2)}$ and this is the expected payment to the bidder. So the expected price will be $E[p] = E[C] + E[c^{(2)} - c \mid c < c^{(2)}]$. Again the possibility of bankruptcy has no effect on the bids or the expected price, it affects only the allocation (probability of bankruptcy). If there are two units for sale the highest signal bidder will quit if she is indifferent between winning or being the marginal bidder (tie with $c^{(2)} = c^{(3)}$). So she will quit if the second highest bidder has the same signal and the expectation of the remaining signal is lower. So the price will be $p = c^{(3)} + c^{(3)} + E[c \mid c \leq c^{(3)}]$ and the expected price will be $E[p] = E[C] + E[c^{(3)} - c \mid c \leq c^{(3)}]$. Again the possibility of bankruptcy has no impact on the price but the allocation rule raises the price and therefore lowers the probability of bankruptcy (see multisourcing).

More interesting is the asymmetric case were we follow Bulow and Klemperer (2002) and assume that Δ_1 is small enough (or ρ_1 is large enough) that bidder one almost always wins. So $\overline{c} + \Delta_1 < c^{(2)} + \Delta < c^{(3)} + \Delta$. Then bidder 3 quits at $c^{(3)} + c^{(3)} + \overline{c}$ and bidder 2 quits at $c^{(2)} + c^{(3)} + \overline{c}$, which is also the price. The advantaged bidder will (almost) always win and so the expected price will be $E[p] = E[C] + E[\overline{c} - c(= c_1)]$. The intuition is that bidder 1 is so advantaged that bidders 2 and 3 face enormous winners curses if bidder 1 exists, and so they have to assume the best case $c^{(1)} \approx \overline{c}$, whenever bidder 1 bids. So they quit at $c^{(2)} + c^{(3)} + \overline{c}$ and bidder 1 (almost) always wins. Leading to a higher payment and a lower probability of bankruptcy compared to the symmetric setting. In this case there is no incentive for the agency to reduce the asymmetry between the bidders, if the agencies B is high enough, because that would increase the probability of bankruptcy. Selling two shares instead of one unit in the symmetric case is reducing the probability of bankruptcy

and for high B will increase the utility of the agency. But with asymmetric bidders this is no longer the case, because selling two shares of a contract creates competition between bidders 2 and 3 for the second share and because they do not compete against bidder 1 for share one, they face a smaller winners curse. And therefore they will bid more aggressive. Because of the increased competition bidder 1 will have to bid lower and if her signal is high enough, she will exit earlier. This further reduces the winners curse for the other bidders. So the expected price in this setting will be $E[p] = E[C] + E[c^{(3)} - c \mid c \leq c^{(3)}]$, which is lower than for the whole unit setting. And therefore an agency (with a high B) might not want to do this, because it increases the probability of bankruptcy.

v) *optimal mechanism*

If the agency can observe and verify the costs of the bidders after the auction, the optimal mechanism would be to pay the difference between the price and the realized cost if the latter are higher. So the optimal mechanism needs bids that are contingent on the realization of the costs. Because bidders ignore more or less the bad state of nature in their bidding behavior (because the profit is zero), a transfer from the agency that also yields zero profit doesn't change their bidding function. So the contract would be the following: Each bidder would submit a bid schedule contingents the price for each realization of the costs. But such contracts can be difficult to implement because of reasons given in the incomplete contract literature (see Hart and Holmstrom (1987)). For instance writing and enforcing a contract like this may be too costly.

3 Other Procurement Mechanisms

Here we compare three different procurement methods and find an understanding of the interaction of the different parameters and give some implications for choice of the "right" mechanism. A motivation for this section is also the search for better procurement mechanisms, as mentioned in section 1. For an easier comparison of the different mechanisms we will use the uniform distribution in this section.

(i) *average bid method*

The average bid method was introduced in some countries to cope with the abnormally low bid problem and it tries to avoid too aggressive bidding and high bankruptcy prob-

abilities. The derivation of the bidding strategy in this format follows the logic of the well known "guess the two third" game, which is the following. There a n participants in a game and each participant can choose a number between 0 and 100. The winner is the participant who chooses the number that is closest to 2/3 of the average and will be awarded a price of say \$10. Before picking a number participant i will think about which numbers the other $(n-1)$ participants will choose, because she needs to calculate the average. So in a first step participant i will think that the average will be around 50, and 2/3 of 50 is $33\frac{1}{3}$. So choosing $33\frac{1}{3}$ would be a good idea. But then she thinks that all the other participants will think that way and therefore the average will be $33\frac{1}{3}$, and 2/3 of $33\frac{1}{3}$ is $22\frac{2}{9}$. So in a second step she would like to choose $22\frac{2}{9}$, and so on until the average will be zero. Any rational participant will behave that way. The equilibrium in the "average bid method" can also be derived the same way by iterated elimination of dominated strategies, leading to a price of $\overline{c} + \Delta$ or higher. The average cost for the uniform distribution is $E[c] = \frac{\overline{c}-\underline{c}}{2}$, neglecting the error term for an easier proof now (with the error term the average would be an additional $(1-\rho) \times \Delta$ higher). Any bid b_i will be c_i or higher, no one will bid below her cost. So the average bid will be $E[c]$ or higher. Now the bidder with the lowest cost can adjust her bid (while the one with the highest cost can not reduce it). She will certainly bid as close to the average as she can, raising the average itself. So will do the second lowest bidder, again raising the average bid, and so on. Because the high cost types will not adjust their bids and will not bid below their costs, the average will increase until it reaches $\overline{c}$ or any price higher if bidders are allowed to bid above $\overline{c}$. We think a maximal willingness to pay of $\overline{c}$ is a reasonable assumption and that is an upper bound of the price. In the end all n bidders will submit the same bid and the winner will be drawn randomly. Once the average is $E[c] = \overline{c}$ we now have to check if no one has an incentive to shade her bid below $E[c] = \overline{c}$. If say the bidder with cost $\underline{c}$ deviates by bidding ϵ below $E[c] = \overline{c}$ then the new average will be $E'[c] = \frac{n*\overline{c}-\epsilon}{n}$. She will win the contract if she is the closest to the average and if the distance is the same she will be awarded the contract because her bid is lower. For a profitable deviation the following inequality must be fulfilled

$$|\overline{c} - \epsilon - \frac{n * \overline{c} - \epsilon}{n}| \leq |\overline{c} - \frac{n * \overline{c} - \epsilon}{n}| \tag{51}$$

The left hand side measures the difference between the bid of the deviating bidder and the new average, and the right hand side measures the difference between the bids of the non-deviating bidders and the new average. So deviating is profitable if the absolute value of the left hand side is smaller or equal than the right hand side, which is only true for $n \leq 2$. So for sufficiently high competition the average bid method will lead to the price of $p = \overline{c}$ and the winner will be drawn randomly, which is not efficient. If we add the error term, the price will be $p_{AB} = \overline{c} + \Delta$ and the bankruptcy probability is again zero ($\phi_{AB} = 0$), but again at a very high price. Small modifications of the allocation rule (say the bid 10% below the average is declared the winner) can reduce the prices, but prices will still be very high.

(ii) *Rationing (truncated English auction)*
Instead of using a SPSB-auction or an English auction the agency might use the following truncated English auction with rationing. Do an English auction until m bidders are left (with $m \leq n$). Then make a lottery between the remaining m bidders where everyone obtains the contract with probability $\frac{1}{m}$. As one extreme case, consider $m = 2$. This implies that the auction stops at $c^{(3,n)}$. The winner is one of the two bidders with the lowest expected costs, however the price ($p_{TE} = E[c^{(3,n)}]$) is higher than under the standard English auction. The probability of bankruptcy is

$$\phi_{TE} = (\frac{1}{2}Prob[c^{(3,n)} - c^{(1,n)} < \Delta] + \frac{1}{2}Prob[c^{(3,n)} - c^{(2,n)} < \Delta])(1 - \rho) \tag{52}$$

which is lower than in a SPSB-auction. As another extreme, consider $m = n$. This implies that the government sources at the price $p = \overline{c} + \Delta$ and makes a lottery between all bidders. This will lead to no bankruptcy, but at a very high price. Depending on the valuation v and on B those mechanisms might fare better than any standard auction. For the two bidder case the allocation is nearly efficient. But in the extreme case the allocation is very inefficient. As happend in the US where the FCC allocated the telecommunication rights until 1993 via lotteries. The inefficient allocations led to different designs, starting the success of auctions in allocating rights.[31]

(iii) *Multisourcing*

[31] Milgrom (2004) p. 3, 79

Assume that the government can split the contract, i.e. it can allocate the contract to two or more bidders. For simplicity, assume that the sourcing is via a SPSB-auction. If the government intends to use two equal sources, the contract goes to the two firms with the lowest bids, they both have to pay the third lowest offer. In this scenario, bidding c is again a dominant strategy. Therefore, the expected price in this framework will be $E[c^{(3,n)}]$. And the probability of bankruptcy will be: $\phi_M = (1-\rho)^2 Prob[c^{(3,n)} - c^{(1,n)} < \Delta]$[32], because we assume that if the bidder with the second lowest costs goes bankrupt the remaining bidder will complete the whole contract if she stays solvent. So the probability above is the probability that both bidders go bankrupt. If the allocation is different, say 70/30, then the price is $p_M = 0.6E[c^{(3,n)}] + 0.4E[c^{2,n}]$ and the probability of bankruptcy is $\phi_M = (1-\rho)^2 Prob[0.6c^{(3,n)} + 0.4c^{(2,n)} - c^{(1,n)} < \Delta]$. So different shares lead to different outcomes and the agency can vary the shares depending on their efficiency goals and bankruptcy costs. The example above showed the effect of multisourcing because its uses the bid distribution to lower the probability of bankruptcy. But also incentives to bid less aggressive are given by multisourcing. Gilbert and Klemperer (2000) have shown that multi sourcing may be preferred to single sourcing in an environment that has different future demand states and requires investment by the bidders. If there are costs of entering an auction a precommitment that allows profits (which is the case when multisourcing) can be desirable because it gives high cost bidders an incentive to participate and that increases incentives for innovation. In a common value environment Bulow and Klemperer (2002) have shown that rationing can increase revenues. If bidders are asymmetric the second source gives the disadvantaged bidders a higher incentive to participate (esp. in a sealed auction, see common value section). So a slightly different environment or allocation rule can change the results dramatically.

Comparison between the procurement methods

Here we compare three different mechanisms ignoring costs of bankruptcy B. Because the FPSB-auction is dominated by the SPSB-auction in terms of bankruptcy probability we will only investigate the SPSB-auction. And because the result of the average bid method and the lottery between n bidders is the same we will only investigate the latter. A lottery leads to no bankruptcy but the price will be $p_R = \overline{c} + \Delta$. So the utility of the

[32] Which is $\phi_M = (1-\rho)^2[1-(1-\frac{\Delta}{\overline{c}-\underline{c}})^n - \frac{n\Delta}{\overline{c}-\underline{c}}(1-\frac{\Delta}{\overline{c}-\underline{c}})^{(n-1)}]$

procuring agency will be: $u_R = v - (\overline{c} + \Delta)$.

The second mechanism is the above described multi-sourcing with two equal shares. The expected price will be $E[p_M] = \frac{3\overline{c}+n\underline{c}-2\underline{\underline{c}}}{n+1}$.[33] The expected utility of the procuring agency will then be: $u_M = (v - E[p_M])(1 - \phi_M)$

The third mechanism is the SPSB, with the results derived above. The expected price will be $E[p_{SPSB}] = \frac{2\overline{c}+n\underline{c}-\underline{\underline{c}}}{n+1}$ and the probability of bankruptcy will be ϕ_{SPSB} from equation (5). The utility of the agency will then be: $u_{SPSB} = (v - E[p_{SPSB}])(1 - \phi_{SPSB})$.

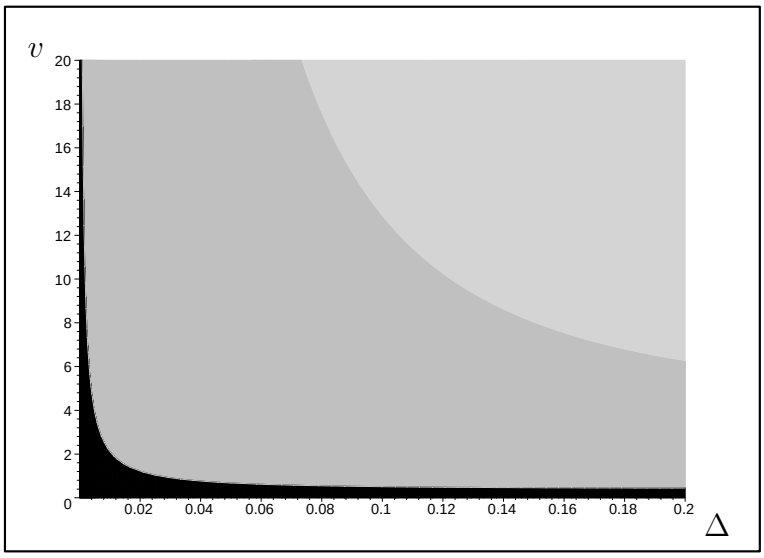

Figure 5: Comparison between the mechanisms for $n = 10$ and $\rho = 0.5$.

Figure 1 shows that the SPSB-Auction (black) fares better, when valuation v and Δ are small. The extreme Rationing method (grey(light)) is better, when the valuation v and Δ are large. And the Multi-Sourcing result (grey) lies in between those two. The interpretation of the value v of the procurement agency can also include opportunity costs of bankruptcy. With high values the costs of bankruptcy become severe so the agency will want to change the mechanism to one that induces less bankruptcy (multisourcing and rationing). This is also more likely the goal of an agency maximizing social welfare (governments etc.). On the other side an agency with low opportunity costs can use the more aggressive bidding due to the bankruptcy possibility to increase her utility. This is more likely the goal of a revenue maximizer, for instance in the private sector. The

[33] This is the expectation of the third lowest order statistic. For the derivation of this result see Wambach (2002).

trade-off in the choice of the agency is to pay informational rents on the one side (low bankruptcy) and opportunity costs on the other side (high bankruptcy).

4 Conclusion

Still to be written ...

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