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Asymmetry and Collusion in Sequential Procurement: Large Lot Last

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Gian Luigi Albano[1] and Giancarlo Spagnolo[2]

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Abstract

Sequential procurement of multiple contracts/lots is widespread and is often regularly repeated. This simple model analyzes how sequentiality and its interaction with asymmetries across bidders and lots affect sustainability of supplier/bidder collusion. Sequential procurement stabilizes collusion (relative to simultaneous procurement) by allowing to identify/punish deviations within the sequence and to allocate the "last lot" to the bidder with the highest incentive to defect. We then analyze how to counteract these effects exploiting (or creating) asymmetries in the value of lots, finding that the most effective policy procures the most valuable lot at the end of each sequence.

Keywords: Bidding rings, Cartels, Collusion, Procurement.

J.E.L. Codes: C7; D4; L4.

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1. Introduction

Multiple contracts (lots) for the supply of similar goods or services are often procured sequentially, at short distance one from the other. Sometimes this is done to simplify the awarding procedure (awarding many lots simultaneously is considered too complex and costly for small size routine sourcing by many procurement managers). Sometimes this is done to reduce the risk of mistakes and legal action, particularly in Public Procurement. These sequences, however, are also often regularly repeated in time, as is typical for most forms of procurement.

Contracts for the supply of different but related goods - say printers, laptops, desktops, monitors, servers - are also often auctioned off sequentially, and for multiproduct suppliers active on several of these goods they are also sequential multi-lot procurement auctions regularly repeated in time.[5]

This short paper discusses simple reasons why recurrent sequential procurements of multiple lots may facilitate bidder collusion relative to simultaneous ones, and identifies an equally simple policy to contain this problem based on (natural or induced) asymmetries among lots and their place in the procurement sequence.[6]

A first familiar reason why sequential procurement may facilitate collusion is that it allows ring members to identify defections from collusive strategies and react to them within a sequence, reducing gains from defecting. The idea that "fragmentation" facilitates cooperation, for the good or for the bad was already stressed by Schelling (1960), formalized by Admati and Perry (1991) and Neher (1999) for joint projects and stage financing respectively, and in a procurement auctions context by Snyder (1996) (see also the informal discussion in Klemperer 2002).

A second, less familiar reason is linked to possible asymmetries between cartel members. The viability of a collusive ring, as that of other types of cartels, is typically limited by "mavericks", bidders with higher expected gains from undercutting the ring (e.g. Baker, 2002). When asymmetric members of a bidding ring share available lots and compensating transfers are risky, the ring can facilitate collusion by allocating to the most aggressive bidder the last lot in the sequence. This reduces the maverick's incentive to defect and stabilizes the ring.

[3] Sequential *sale* auctions of multiple objects or lots are also frequent in electricity, timber, tobacco and oil lease markets, where they are may also be repeated in time. Recurrent sequences of auctions, though common, appear not to have been studied by the literature so far, and to be particularly subject to bidder collusion. In the US Tobacco market, for example, sellers recently changed from a sequential open format to a sequential sealed bid first price format because bidders were perceived to collude. Government securities and mineral rights, on the other hand, are also sold repeatedly, but the multiple lots are generally sold in simultaneous auctions. We are grateful to Peter Cramton for suggesting all these examples.

[6] In the reminder of the paper we will use the words "bidder" and "supplier" as well as "lot" and "contract" as synonyms. Moreover, although our results apply also to repeated sequences of sale auctions, we will maintain focus on procurement as the interaction between suppliers is often so frequent and stable in time that preventing collusion becomes the single main problem for the procurer.

Both pro-collusive effects may interact with the asymmetry in the lots sold and with their place in the sequence. After characterizing the role of lot asymmetry, we look for policies to fight bidder collusion. We show that if turning to a simultaneous setting is too costly, a simple and effective strategy against collusion is a "large lot last" policy - i.e. tendering the most valuable lot at the end of each sequence, so that the largest deviation cannot be punished before a new sequence of procurement starts, which may happen much further in the future. Since the value of each of the lots procured and their place in the sequence are typically decisions in the hands of the procurer, implementing such a policy appears rather easy.

Our paper focuses on collusion in procurement auctions and on policies to prevent it, a literature partly surveyed in Klemperer (2002). In particular, our simple model with complete information across bidders is in the tradition of papers like Wilson (1979), Robinson (1985), Snyder (1996) and, at least in part, Cramton and Schwartz (2000), as they also identify likely pro-collusive effects of different competitive procedures that do not depend on informational variables. like disclosure policies, communication or signalling.[5]

Section 2 sets up a simple model and derives benchmark results; Section 3 introduces lot asymmetry and derives the collusion-preventing policy; Section 4 briefly concludes. Proofs are in an Appendix.

[5] Models dealing with asymmetric information across bidders are much more complex. See for example McAfee and MacMillan (1992), Hopenhayn and Skrzypacz (2004) and Marshall and Marx (2007), among many others.

2. Set up and benchmarks

We adopt the simplest reduced form model we could conceive, as the forces we identify are intuitive and mechanical and therefore likely not to depend much on the specific formulation of the market game. In a more complex environment with imperfect information those forces should remain active and interact with other relevant forces, which may or not outweigh them.

There are two "relevant" long-lived potential suppliers, 1 and 2, among which the relevant characteristics (production costs for performing a procurement contract, discount factors, etc.) are common knowledge.[6] The simplifying assumption of complete information between bidders allows us to better focus on the effects of sequentiality and asymmetry and is not too unrealistic for many procurement situations, particularly in mature markets where suppliers are long-term competitors with a lot of personnel exchanges/turnover and knowledge of the market.

Two procurement contracts (say, geographical lots), named *A* and *B*, are awarded through competitive procurements, say first price sealed bid auctions, in every period t, $t = 1, 2...$, either sequentially or simultaneously within a negligible time frame.
Suppliers discount time according to discount factors $0 < \delta_1, \delta_2 < 1$. We assume for simplicity and w.l.o.g. no discounting between the first and the second lot of a given sequence when procurement is sequential.

2.1 The fully symmetric benchmark case

Suppose first that bidders are identical, namely that $\delta_1 = \delta_2 = \delta$ and that $c_1 = c_2 = c > 0$. That is, we temporarily assume here that suppliers have the same discount factor and bear the same production cost for undertaking both procurement contracts.
Assume also that the procurer commits to the same publicly announced reserve price, r, for each contract, with $r > c$. We find convenient to define $\overline{v} \equiv r - c$ as the upper bound to suppliers' profit from each procurement contract. The two contracts are repeatedly awarded through either a simultaneous or a sequential tendering format. In the latter case, all information is made public at the end of each round.

[6] The two firms could alternatively be considered as two "dominant firms" in presence of a fringe that has little chances to win; see Albano and Spagnolo (2005).

Since suppliers are identical and there is an upper bound on profits, the short-run competitive equilibrium in a one-shot competitive tendering is the Bertrand solution, delivering zero profits to each supplier (one can define these 0 extra profits, 0 being a normalization for the profits that can be earned from other activities). Suppose, further, that when suppliers decide to collude they support the ring by the threat of reverting to competitive behavior forever in case a deviation is observed. This punishment is optimal in this section's symmetric environment as it minimaxes the deviator, exactly as in a symmetric Bertrand supergame.

2.2 Split-award collusion

Suppose suppliers agree to collude by splitting each period, today and in the future, the two lots at the reservation price *r*. Sustaining split-award collusion in the future delivers $\overline{v}$ as net collusive profit to each supplier in every period, so that the discounted expected stream of collusive profits is $V^C = \frac{\overline{v}}{1 - \delta}$, independent of whether a simultaneous or sequential procurement auctions are used. As mentioned earlier, gains from defection (V^D) typically depend on whether the procurement auction used to allocate lots is sequential or simultaneous, because with sequential auctions opponents observe and can react earlier to the defection. However, in a symmetric environment this is only true if there are more than two lots to allocate. In fact, we have the following.

Lemma 1. *In a fully symmetric environment with two lots awarded repeatedly and two bidders, the condition at which a stationary split-award collusive agreement is sustainable in equilibrium does not depend on whether the stage-game procurement auction is simultaneous or sequential.*

Both when the format is simultaneous and when it is sequential, the maximum gains from undercutting obtainable with the split award collusive agreement is one more lot at a price marginally below the agreed cartel price. So in both cases collusion is supportable in equilibrium if

$$V^C = \frac{\overline{v}}{1 - \delta} \geq 2\overline{v} = V^D \Longleftrightarrow$$

$$\delta \geq \delta^{Split} = 1/2,$$

and the lemma's irrelevance result follows.

2.3 Bid rotation

What if suppliers agree instead on a bid rotation scheme, whereby they take turn in getting both contracts A and B simultaneously at the reservation price? Consider the incentives of any of the two suppliers to defect in a period when, according to the bid rotation scheme, the other supplier should win both objects. If the former does not defect, it earns zero profits today but it expects discounted future profits $V^C = \delta \frac{2\bar{v}}{1-\delta^2}$. If it defects, in the future it will earn zero, but short run gains from defection *do* depend on which type of competitive format is adopted.

If the format is simultaneous a defecting supplier can earn $2\bar{v}$; if it is sequential,instead, it can only earn $\bar{v}$, as the opponent can observe the defection and react competing immediately on the second contract of the same sequence. Alternatively, and analogously, the defecting supplier might defect only on the second contract of the same sequence earning the same profit $\bar{v}$ and triggering Bertrand competition from the next stage afterwards.

Hence, with a sequential format the incentive constraint for a bid rotation scheme being sustainable in equilibrium is

$$V^C = \delta \frac{2\bar{v}}{1-\delta^2} \geq \bar{v} = V^D \iff \delta^2 + 2\delta - 1 \geq 0 \iff$$

$$\delta \geq \delta^{RotSq} = \sqrt{2} - 1 < 1/2 = \delta^{Split},$$

and for the simultaneous auction is

$$V^C = \delta \frac{2\bar{v}_i}{1-\delta^2} \geq 2\bar{v} = V^D \iff \delta^2 + \delta - 1 \geq 0 \iff$$

$$\delta \geq \delta^{RotSm} = \frac{\sqrt{5}-1}{2} > \delta^{Split} > \delta^{RotSq}.$$

This reasoning leads to the following:

Proposition 1. *In a fully symmetric environment with two lots awarded repeatedly and two suppliers, the condition at which a bid rotation collusive agreement is sustainable in equilibrium is more stringent if the stage-game procurement format is simultaneous than if it is sequential ($\delta^{RotSm} > \delta^{RotSq}$). Moreover, the condition at which a bid rotation equilibrium is sustainable is more stringent than the one at which a correpondent split award agreement is sustainable if a simultaneous procurement auction is used, whereas the opposite is true if a sequential one is used ($\delta^{RotSm} > \delta^{Split} > \delta^{RotSq}$)*

In other words, a bid rotating scheme together with a sequential auction re-activate Schelling's "fragmentation" effect even in a 2-lot-2-supplier environment, so that a bid rotation scheme becomes easier to sustain than a split the contracts scheme. The converse is true if a simultaneous auction is used.

Remark: *The results suggest that, if colluding bidders can choose among splitting the contracts and bid rotation and the discount factor is binding, then they will choose to split the contracts if the procurement auction is simultaneous, and bid rotation when the procurement auction is sequential.*

2.4 Asymmetric bidders and the 'Maverick' effect

The assumption of perfect symmetry between suppliers is admittedly restrictive. In this section we assume that the two bidders are asymmetric in terms of their incentives to collude, so that with a symmetric allocation of the collusive outcome one of the bidders' incentive compatibility constraint is violated and the other is slack. Suppliers might differ in capacities, hence ability to gain from deviations and punish, as in Comte *et al.* (2002); or, they can be different in costs, as in Miklos-Thal (2010); or else they can differ in their long-run fitness to survive in the market or access to financial resources, hence in discount factors. Whatever the source of the asymmetry, one of the two suppliers plays the role of the "maverick", as the latter's stronger incentive to defect limits the ability of the two suppliers to collude (see Baker 2002).

Suppose, further, that suppliers face transaction costs in redistributing collusive gains across the two bidders, through transfers or randomization schemes, to pool the two incentive constraints (for example because of the complexity of the redistribution schemes, or of the higher risk of being detected by the competition authority when operating such transfers). Then even at the optimal collusive scheme the incentive compatibility constraints of the two asymmetric bidder will not be identical, and one of the two suppliers will continue to play the role of a maverick that constraints their joint ability to sustain collusion.

We show here that in the split-award collusive agreement a sequential tendering format allows suppliers to eliminate the maverick effect by allocating to the maverick the last contract awarded within the same sequence. That is, if a sequential format is used instead of a simultaneous one, bidders can strictly improve on the optimal split-award collusive scheme by allocating the second lot in the sequence to the maverick and the first one to the supplier with a slack incentive constraint. Such an allocation scheme, by softening the maverick's incentive constraint, makes collusion easier to sustain. In fact, it turns out that this effect induces suppliers to prefer a split-award collusive scheme to a bid rotation one. The result is in stark contrast with what found under full symmetry whereby the bid rotation is more effective than split-award (since, together with a sequential format, it revives Schelling's "fragmentation" effect).

Proposition 2. *When bidders are asymmetric in terms of incentive compatibility constraints and transfers are costly, so that there is always a 'maverick' bidder that constraints collusion, choosing a split-award arrangement and allocating the last lot to the maverick facilitates collusion if the procurement auction is sequential.*

The simple policy conclusion we can draw from this is that a buyer should try *not* to use a sequential auction, and prefer instead a simultaneous one, when i) the risk of collusion among bidders is nonnegligeable; ii) the auctioned contracts are similar in value, but iii) bidders are rather asymmetric in dimensions that may aff ect their willingness to stick to a collusive agreement.

3. Lot asymmetry and the LLL policy

Suppose now for simplicity that bidders are symmetric, as assumed in Section 2, but that lots are or can be designed to be heterogenous in value. That is, assume that the lots A and B have a different value for bidder, namely (w.l.o.g.) $\overline{v}^A > \overline{v}^B$ and $\overline{v}^A + \overline{v}^B = 2\overline{v}$. Again, an optimal collusive scheme would aim at designing and implementing transfers between suppliers in order to re-allocate collusive gains symmetrically. However, as already discussed in the previous section, direct monetary transfers between competitors are complex to administer and substantially increase the likelihood of a conviction by the competition authority. Equilibrating transfers between cartel members, however, could be operated intertemporally, either through a full bid rotation scheme in which suppliers take turn in winning both lots at a collusive price; or by a partial rotation split award collusive scheme in which suppliers split the lots in each period but take turns in receiving the more valuable one. We show here that when a sequential auction is used, it is more difficult for bidders to collude if the more valuable lot is always auctioned at the of *end* the sequence.

3.1 Simultaneous Procurement

Consider first the case in which a simultaneous procurement auction is used. Suppliers can still choose between a full bid rotation scheme, in which suppliers take turns winning both contracts, and a split-award scheme where suppliers alternate in getting the larger lot *A*. It is easy to verify that with *a full rotation scheme* suppliers' incentive constraints coincide with those in the case of symmetric lots analyzed in Section 2: short run gains from defection are the sum of lots' value, which is still $2\overline{v}$; the punishment phase is exactly like in Section 2. Hence, full collusion with full bid rotation is sustainable if

$$V^C = \delta \frac{2\overline{v}}{1-\delta^2} \geq 2\overline{v} = V^D \Longleftrightarrow \delta^2 + \delta - 1 \geq 0 \Longleftrightarrow$$

$$\delta \geq \delta^{RotSm} = \frac{\sqrt{5}-1}{2}.$$

Consider now a partial rotation *split award collusive* scheme. The more profitable defection consists in any supplier 'stealing' the more valuable contract in a period in which the agreement allocates her the less valuable one. The expected payoffs from such a defection would be $V^D = \overline{v}^A + \overline{v}^B$, as the payoffs in the following punishment phase are zero. Expected discounted profits from sticking to the collusive strategy, including that period's payoff, are $V^C = \overline{v}^B + \delta \frac{\overline{v}^A}{1-\delta^2} + \delta^2 \frac{\overline{v}^B}{1-\delta^2}$. Hence, a partial rotation split award scheme is supportable via a simultaneous procurement auction iff

$$V^C = \bar{v}^B + \delta \frac{\bar{v}^A}{1-\delta^2} + \delta^2 \frac{\bar{v}^B}{1-\delta^2} \geq \bar{v}^A + \bar{v}^B = V^D \Longleftrightarrow \tag{1}$$

$$\delta \geq \delta^{AsSplSm} = \frac{-\bar{v}^A + \sqrt{(\bar{v}^A)^2 + 4(\bar{v}^A+\bar{v}^B)\bar{v}^A}}{2(\bar{v}^A+\bar{v}^B)} < \delta^{RotSm}.$$

3.2 Sequential Procurement

Consider now the case in which a sequential procurement auction is used. As usual, suppliers can choose between a full bid rotation scheme, in which they take turns winning both auctioned contracts, and a split award scheme whereby they alternate in getting the larger contract *A*.

With a *full bid rotation* scheme suppliers' incentive constraints are slightly different than in the case of identical contracts analyzed in Section 2, as gains from defection vary owing to the asymmetry between lots. Consider the incentives of supplier $i \in \{1, 2\}$ to defect in a period when, according to the bid rotation scheme, supplier $j = 3 - i$ should win both objects. If the supplier does not defect, in earns zero profits today but it expects discounted future profits $V^C = \delta \frac{\bar{v}^A + \bar{v}^B}{1-\delta^2} = \delta \frac{2\bar{v}}{1-\delta^2}$. If it defects, in the future it will earn zero, but short run gains from defection are now $V^D = \bar{v}^A > \bar{v}$. Regardless of whether lot *A* is auctioned first or second, the most profitable deviation consists in undercutting on that lot, after which the punishment starts, as the opponent can observe the defection and react competing already on the second lot of that sequence in case the lot *A* was first. Hence, with a sequential auction the incentive constraint for a bid rotation scheme being sustainable in equilibrium is

$$V^C = \delta \frac{\bar{v}^A + \bar{v}^B}{1-\delta^2} \geq \bar{v}^A = V^D \Longleftrightarrow \delta^2 \bar{v}^A + \delta(\bar{v}^A+\bar{v}^B) - \bar{v}^A \geq 0 \Longleftrightarrow \tag{2}$$

$$\delta \geq \delta^{AsRotSq} = \frac{-(\bar{v}^A+\bar{v}^B) + \sqrt{(\bar{v}^A+\bar{v}^B)^2 + 4(\bar{v}^A)^2}}{2\bar{v}^A} < \delta^{RotSm}.$$

Consider now the crucial case of a partial rotation *split award* scheme with the sequential format. The most profitable defection from the collusive schemes depends on whether the larger lot is auctioned before or after the smaller one. If the more valuable lot (*A*) is auctioned last, the most profitable defection is the one of the bidder who should get lot *B*: in fact, that supplier gets its lot first, and then steals the larger lot *A* undercutting his collusive partner. Collusion is then sustainable only iff

$$V^C = \bar{v}^B + \delta \frac{\bar{v}^A}{1-\delta^2} + \delta^2 \frac{\bar{v}^B}{1-\delta^2} \geq \bar{v}^B + \bar{v}^A = V^D \iff \tag{3}$$

$$\delta \geq \delta^* = \frac{-\bar{v}^A + \sqrt{(\bar{v}^A)^2 + 4(\bar{v}^A+\bar{v}^B)\bar{v}^A}}{2(\bar{v}^A+\bar{v}^B)} = \delta^{AsSplSm}.$$

If instead the large lot is auctioned always at the beginning, then maximum gains from defections are limited to max $\{ \bar{v}^A - \bar{v}^B; \bar{v}^B \}$. In fact, if asymmetry is very strong, so that $\bar{v}^A > 2\bar{v}^B$, then the most profitable short run defection are those of the supplier that should have got lot *B* at the second round but instead decided to undercut at the first round, obtaining extra profits $\bar{v}^A - \bar{v}^B$. Its incentive compatibility condition is then

$$V^C = \bar{v}^B + \delta \frac{\bar{v}^A}{1-\delta^2} + \delta^2 \frac{\bar{v}^B}{1-\delta^2} \geq \bar{v}^A - \bar{v}^B = V^D \iff \tag{4}$$

$$\delta \geq \delta' = \frac{-\bar{v}^A + \sqrt{(\bar{v}^A)^2 + 4(\bar{v}^A-\bar{v}^B)(\bar{v}^A-2\bar{v}^B)}}{2(\bar{v}^A-\bar{v}^B)} < \delta^{AsSplSm}.$$

If instead asymmetry is not too large, $\bar{v}^A < 2\bar{v}^B$, then the maximum short run gains from defection are those of the supplier that according to the collusive agreement obtains *A* first, and then decides to undercut to obtain also *B* at the second stage, obtaining $\bar{v}^B$ as net gains from defection. In this second case collusion is sustainable iff

$$V^C = \bar{v}^A + \delta \frac{\bar{v}^B}{1-\delta^2} + \delta^2 \frac{\bar{v}^A}{1-\delta^2} \geq \bar{v}^A + \bar{v}^B = V^D \iff$$

$$\delta \geq \delta'' = \frac{-\bar{v}^B + \sqrt{(\bar{v}^B)^2 + 4(\bar{v}^A+\bar{v}^B)\bar{v}^B}}{2(\bar{v}^A+\bar{v}^B)} < \delta' \mid \bar{v}^A < 2\bar{v}^B < \delta^* = \delta^{AsSplSm}.$$

The above inequalities allow us to state our main result.

Proposition 3. (Large Lot Last policy) *When bidders are symmetric, lots are asymmetric and a sequential procurement auctions must be used to award them each period, procuring the most valuable lot at the END of the sequence minimizes bidders' ability to collude by making it identical to when a simultaneous procurement auction is used* ($\delta^{RotSm} > \delta^{AsSplSm}$).

The policy continues working when bidders have asymmetric valuations but the asymmetry is small relative to that in lots' value. To see this, one could de ne $\overline{v}_i^k$ as bidder i's maximum profit from competing for contract k. Proposition 3 tells us that when $\overline{v}_1^A = \overline{v}_2^A = \overline{v}^A > \overline{v}^B = \overline{v}_1^B = \overline{v}_2^B$ the "large-lot-last" policy in a sequential format makes collusion as sustainable as under a simultaneous format. However, by continuity, the result is robust under small perturbations such as $\overline{v}_1^A \approx \overline{v}_2^A$ and $\overline{v}_1^B \approx \overline{v}_2^B$, that is, so long as a slight asymmetry arises between bidders' valuations of the same contract. Sufficient asymmetry in lots' value create robustness of this policy relative to asymmetries in bidders' valuation, and can be achieved whenever the procurer can purposely design lots of different value.

4. Discussion and conclusions

Groups of multiple procurement contracts are often and repeatedly awarded over time by means of a sequential competitive auctions. We have shown that the buyer may try to hamper suppliers' attempts to collude by generating and exploiting heterogeneity in lots' value.

When the number of contracts goes up, for a fixed number of bidders, the two procollusive effects bite more under a sequential format. First, bidders are able to exploit a growing number of allocations of lots within the same sequence in order to better align incentive constraints, thus remedying the destabilizing effect of the "maverick" bidder. Second, profits from defection on a single contract become less attractive the longer the sequence of contracts awarded at each date. We see no reasons, however, why a larger number of lots relative to the number of bidder could undermine the anti-collusive effect of tempting members of a ring by procuring the most valuable lot at the end of each sequence.

Arguably, a further dimension of asymmetry may arise. When bidders have opposite ranking of the two contracts - e.g., $\overline{v}_1^A > \overline{v}_2^A$ and $\overline{v}_1^B < \overline{v}_2^B$ - incentives to cooperate and, symmetrically, temptations to defect are not aligned anymore. The characterization of bidder optimal strategies and effective anti-collusive policy is likely to become much more cumbersome. However, situations where bidders' idiosyncratic valuation are highly non-monotone in the value of lots are uncommon in regular procurements, the markets we are focussing on. Also, these are situations in which such valuations are more likely to be private information, making bidder optimal strategies much more complex and a policies conditional on that information have little practical value as the procurer would not know how to apply it. Still, that case would have substantial theoretical interest, so we regard it as a fruitful avenue for future theoretical work.

5. Appendix

Proof of Proposition 2. Suppose that transfer schemes are prohibitively costly, and that supplier 1 is the "maverick", so that the latter's incentive constraint is not satisfied under any of the collusive schemes discussed in the previous section, that is, $\delta_1 < \delta^{RotSq}$. Suppose, instead, that supplier 2 has strong incentives to stick to collusion, that its incentive constraint is satisfied at all the collusive paths discussed in the previous section, that is, $\delta_2 > \delta^{Split} > \delta^{RotSq}$.

If a simultaneous format is used, the collusive agreement is not sustainable because supplier 1, the maverick, would defect both from a split-award agreement ($\delta_1 < \delta^{Split}$) and from a bid rotation one ($\delta_1 < \delta^{RotSq}$). If a sequential format is used, and bidders choose a bid rotation collusive scheme, the agreement is not sustainable because supplier 1 would defect from it ($\delta_1 < \delta^{RotSq}$).

If a sequential format were adopted, and bidders chose instead a split-award agreement, they could also agree that the first auctioned contract in each period to be assigned to supplier 2, and the second auctioned contract to supplier 1, the maverick.

Consider the profitability of defections from this *maverick-last* agreement. Supplier 2 will not defect because we assumed $\delta_2 > \delta_{\text{Split}}$: supplier 1, the maverick, would defect if it were to be assigned the first contract ($\delta_1 < \delta^{RotSq}$), but would gain nothing from defecting if it were allocated the second contract. The only feasible defection is to steal supplier 2's contract. However, if supplier1 does so supplier 2 would revert to Bertrand competition immediately afterwards, so that supplier 1's *additional* gains from defection are zero. Therefore it is strictly better also for the maverick not to defect.

Proof of Proposition 3. We know already that second part of the statement holds since the relevant incentive compatibility constraints (3) and (1) coincide. We have then to prove that the critical discount factor for collusion to be sustainable is highest under the LLL policy, namely that $\delta^* > \delta', \delta'', \delta^{AsRotSq}$. Instead of comparing directly the expressions of each discount factor we will proceed by comparing the relevant incentive compatibility constraints.

i) $\delta^* > \delta^{AsRotSq}$: Consider inequalities (3) and (2). By adding $\frac{(1-\delta)\bar{v}^B}{1-\delta^2}$ to both sides of (2) one can easily see that the left-hand sides (i.e., the discounted payoffs from sticking to the collusive strategies) of both (3) and (2) coincide. We are left then to show that the right-hand side of (3) is always higher than that of (2), that is,

$$\bar{v}^A + \bar{v}^B \geq \bar{v}^A + \frac{(1-\delta)\bar{v}^B}{1-\delta^2} = \bar{v}^A + \frac{1}{1+\delta}\bar{v}^B$$

which is always true since $\delta \in (0, 1)$:

ii) $\delta^* > \delta'$. This is immediate since the left hand sides of both (3) and (4) coincide, whereas the right-hand side of (3) is strictly greater that of (4).

iii) $\delta^* > \delta''$. In this case the right-hand sides of both (3) and (5) coincide. We have to show that the left-hand side of (3) is strictly greater than that of (5), that is,

$$\bar{v}^A + \delta \frac{\bar{v}^B}{1-\delta^2} + \delta^2 \frac{\bar{v}^A}{1-\delta^2} > \bar{v}^B + \delta \frac{\bar{v}^A}{1-\delta^2} + \delta^2 \frac{\bar{v}^B}{1-\delta^2} \Longleftrightarrow$$

$$\bar{v}^A + \delta \bar{v}^B > \bar{v}^B + \delta \bar{v}^A$$

which is always true since $\bar{v}^A > \bar{v}^B$ and $\delta \in (0; 1)$ by assumption.

References

[1] Admati, A.R. and M. Perry, 1991, "Joint Projects without Commitment", *Review of Economic Studies*, 48, 259-276.

[2] Albano G. L., Spagnolo G., 2005, "The Collusive Drawbacks of Sequential Auctions", in *Quaderni Consip* No. IX.

[3] Baker J., 2002, "Mavericks, Mergers, and Exclusion: Proving Coordinated Competitive Effects under the Antitrust Laws", *New York University Law Review*, 77, 135-203.

[4] Compte, O., Jenny, F. and P. Rey, 2002, "Capacity Constraints, Mergers and Collusion", *European Economic Review*, 46, 1-29.

[5] Cramton, P. and J.A. Schwartz, 2000, "Collusive Bidding: Lessons from the FCC Spectrum Auctions", *Journal of Regulatory Economics*, 17, 229-252.

[6] Marshall, R.C. and L.M. Marx, 2007, "Bidder Collusion", *Journal of Economic Theory*, 133, 374-402.

[7] Klemperer, P., 2002, "What Really Matters in Auction Design", *Journal of Economic Perspectives*, 16, 169-189.

[8] McAfee, P.R. and J. McMillan, 1992, "Bidding Rings", *American Economic Review*, 82, 579-599.

[9] Miklos-Thal, J., 2010, "Optimal Collusion under Cost Asmmetry", forthcoming in *Economic Theory* .

[10] Neher, D.V., 1999, "Staged Financing: an Agency Perspective", *Review of Economic Studies*, 66, 255-274.

[11] Robinson, M.S., 1985, "Collusion and the Choice of Auction", *RAND Journal of Economics*, 16, 141-145.

[12] Schelling, T. C., 1960, The Strategy of Conflict, Harvard University Press.

[13] Skrzypacz, A. and H., Hopenhayn, 2004, "Tacit Collusion in Repeated Auctions," *Journal of Economic Theory*, 114, 153-169.

[14] Snyder, C., 1996, "A Dynamic Theory of Countervailing Power", *RAND Journal of Economics*, 27(4), 747-769.

[15] Wilson, R., 1979, "Auctions of Shares," Quarterly Journal Economics, 93, 675-689.

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